Imaging of Construction Materials and Geomaterials
Organized by Chair Materials Science for Sustainable Construction supported by LafargeHolcim
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On the Use of Digital Volume Correlation for the Identification of the Crushing Behavior of Plaster

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PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR
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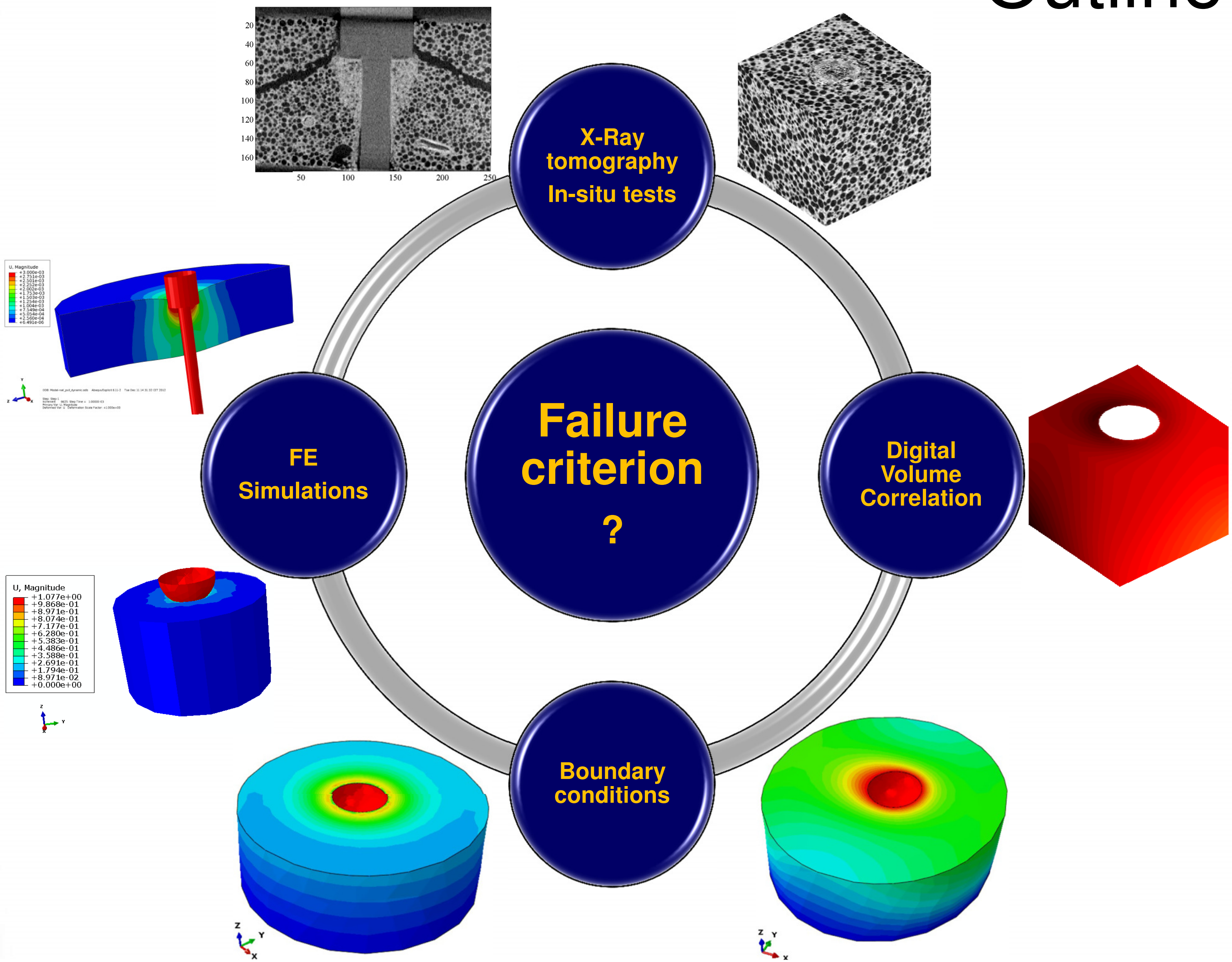
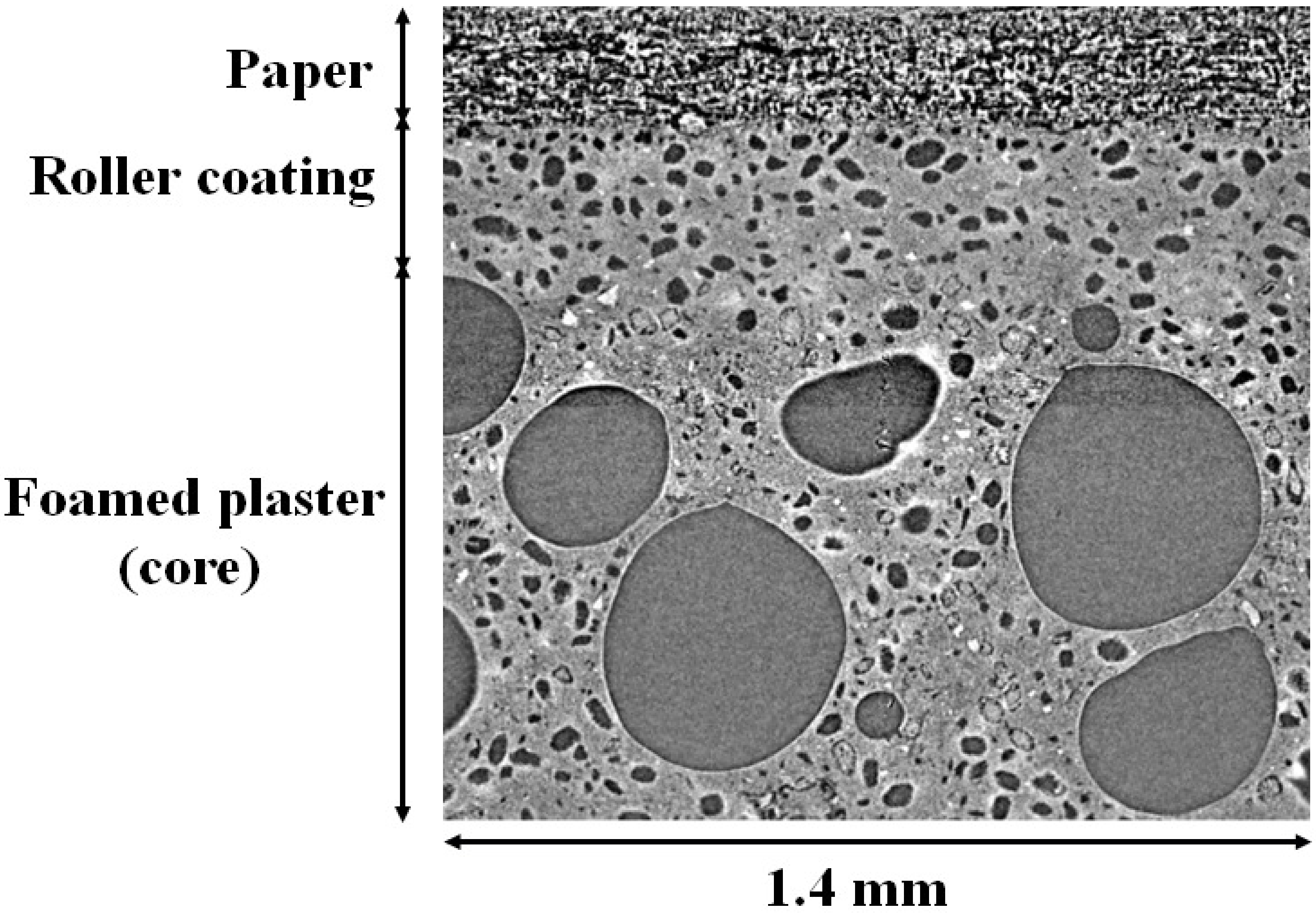
SAINT-GOBAIN RECHERCHE

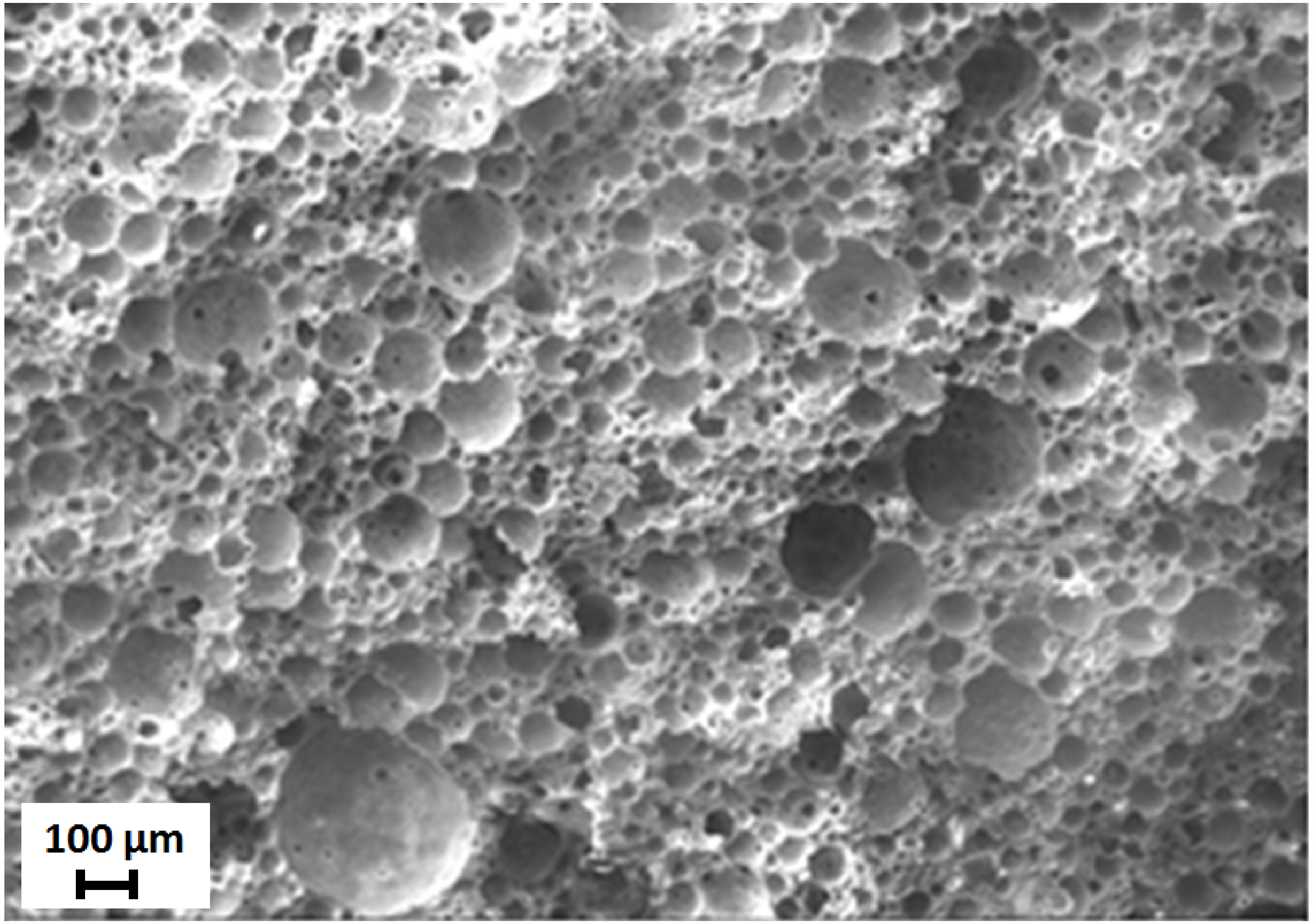
Outline


X-Ray tomography
In-situ tests
Digital Volume Correlation
Boundary conditions
FE Simulations
Failure criterion
?
U, Magnitude
+1.077e+00
+9.868e-01
+8.971e-01
+8.074e-01
+7.177e-01
+6.280e-01
+5.383e-01
+4.486e-01
+3.588e-01
+2.691e-01
+1.794e-01
+8.971e-02
+0.000e+00

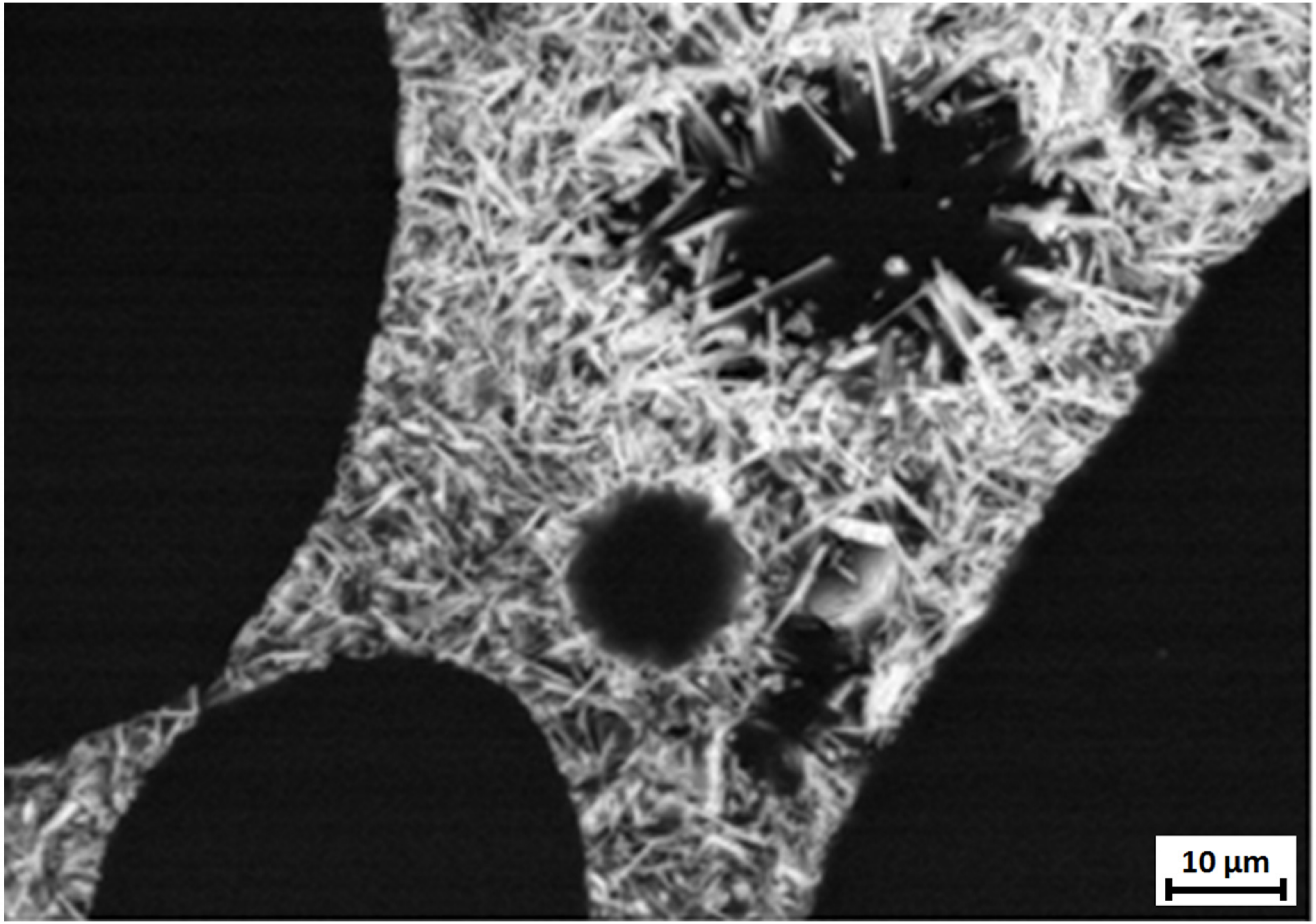
Plasterboard



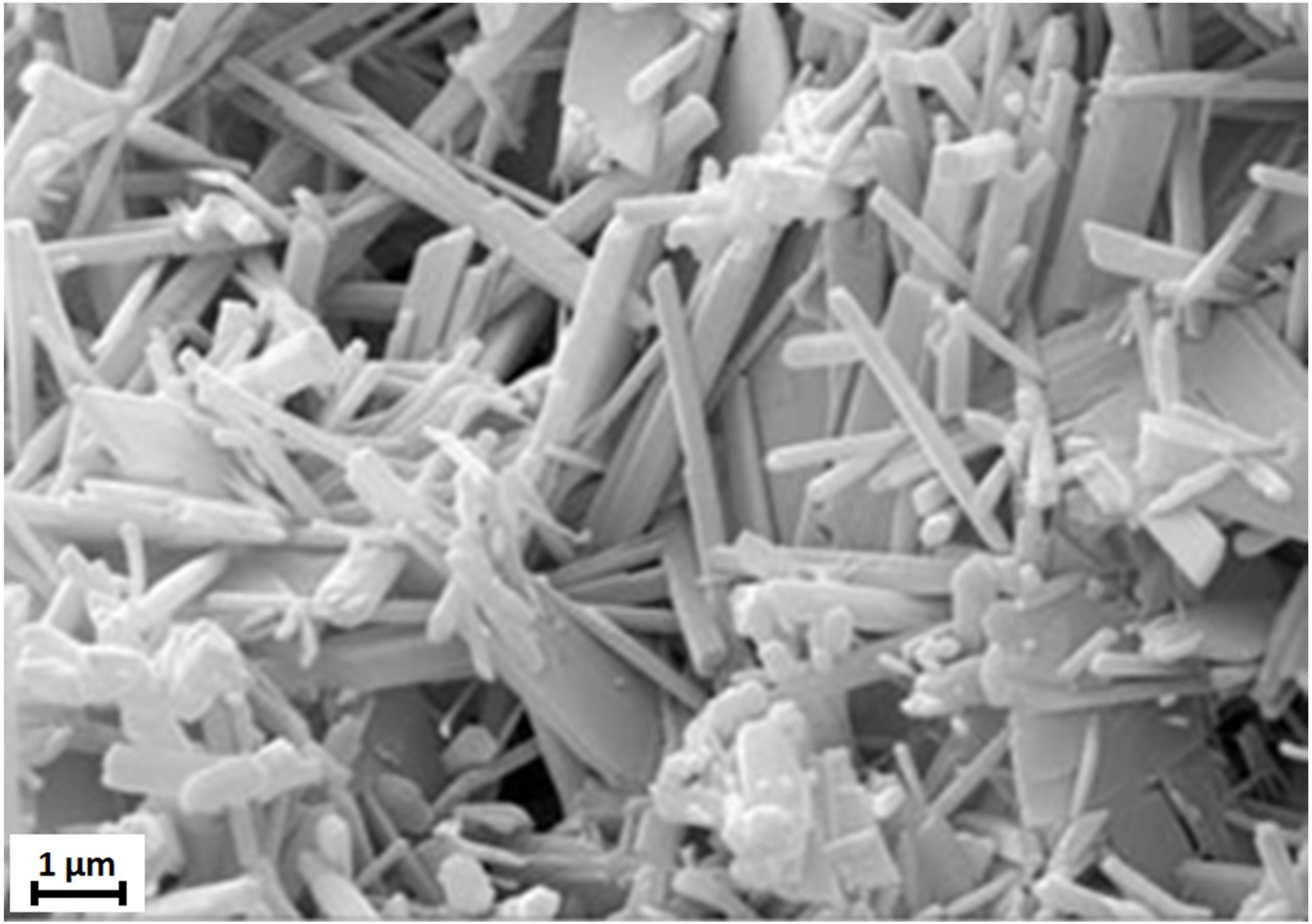
Brittle Foam



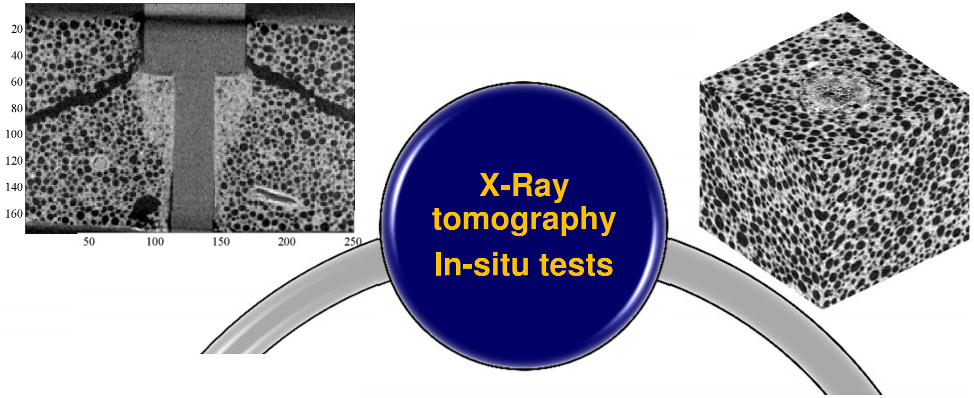
Multiscale Microstructure



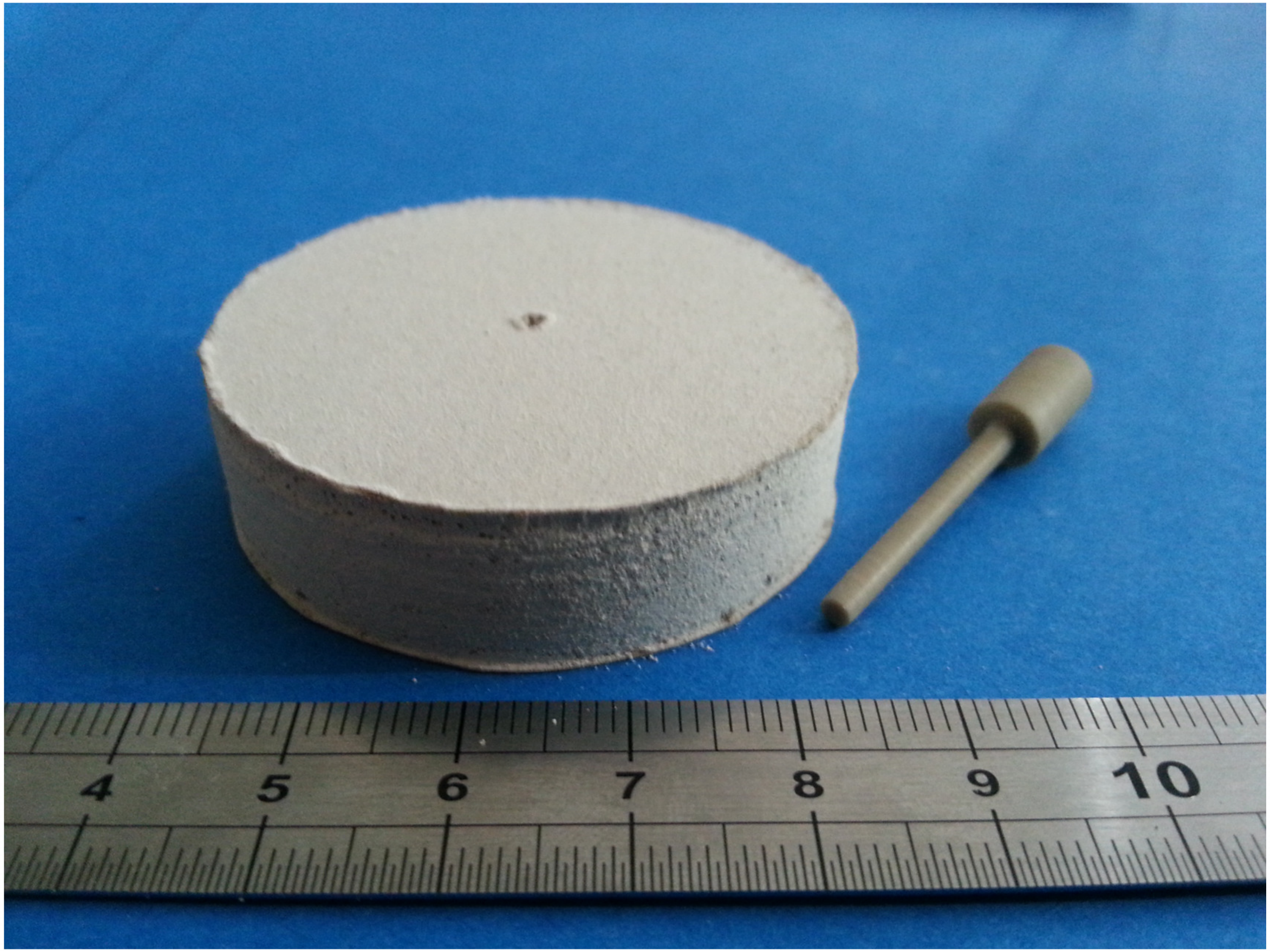
Gypsum



Outline

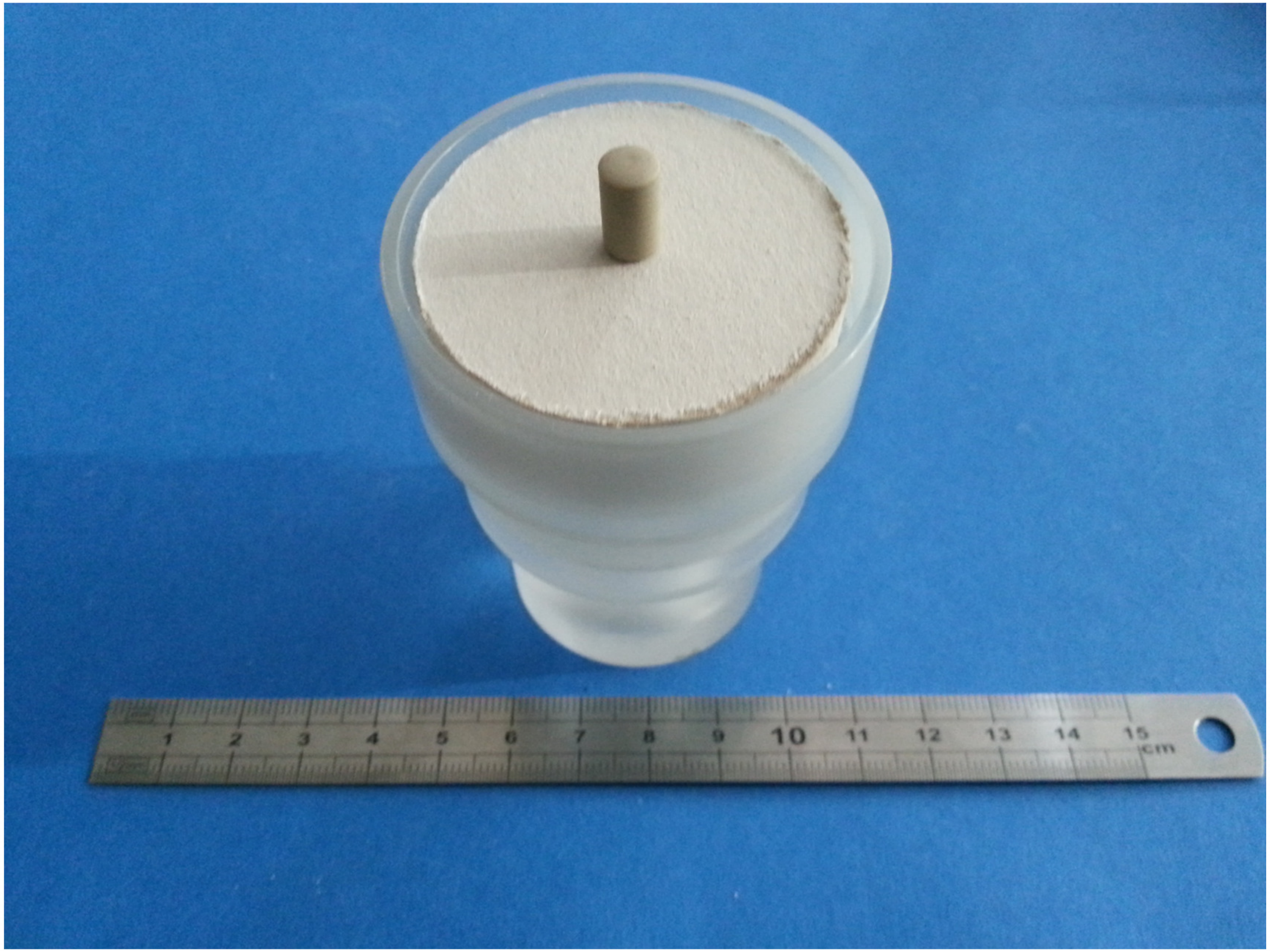


Nail Pull Test on Plasterboard

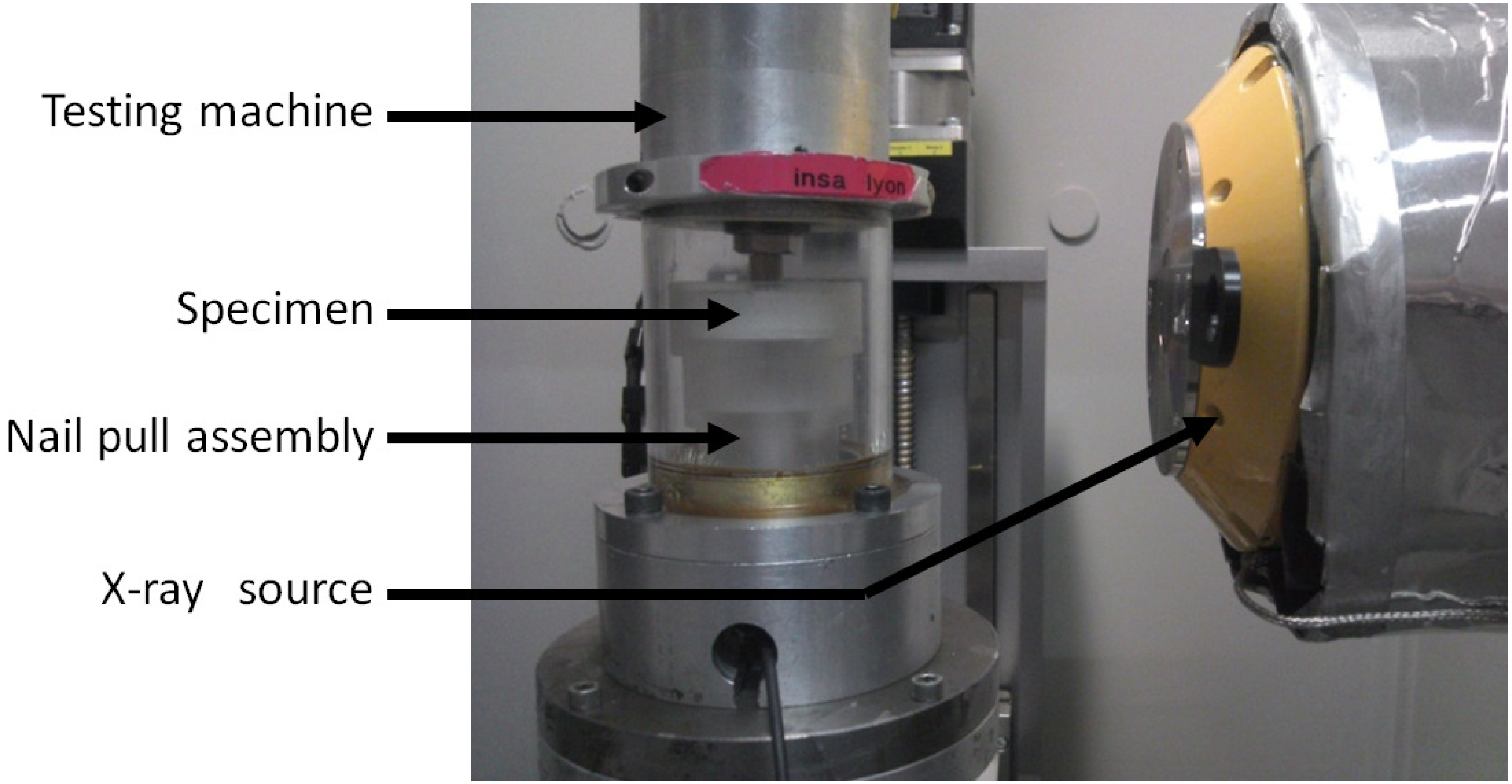


[ASTM Standard C1396/C1396M-14, 2014, DOI: 10.1520/C1396_C1396M]

Nail Pull Test on Plasterboard

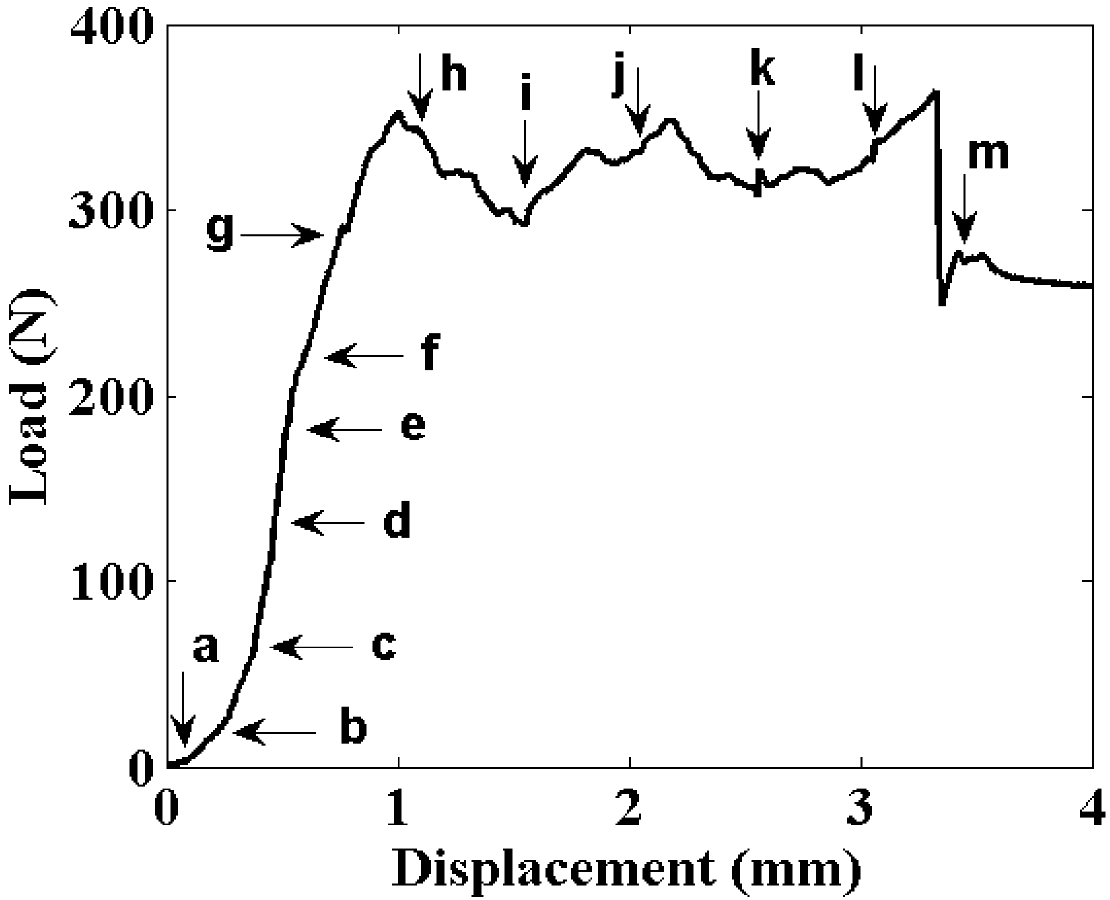


In-Situ Nail Pull Test

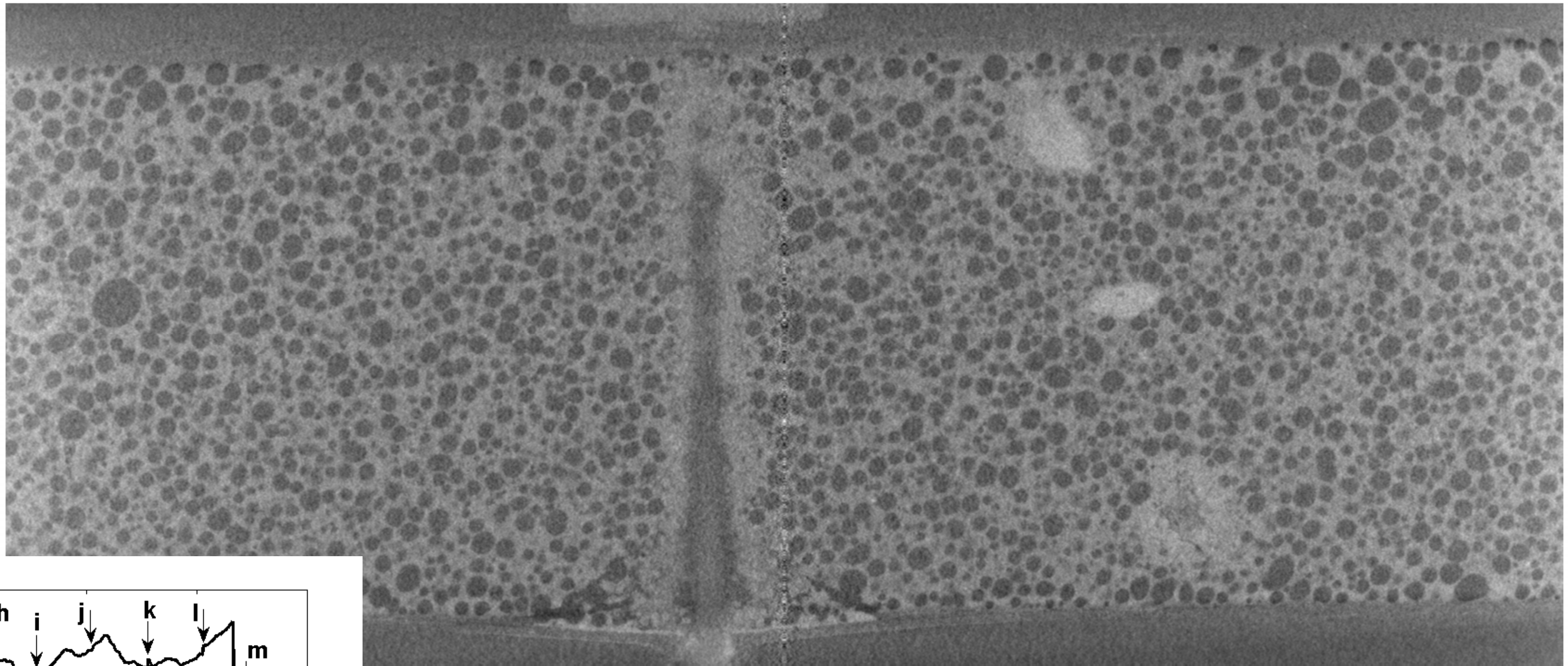


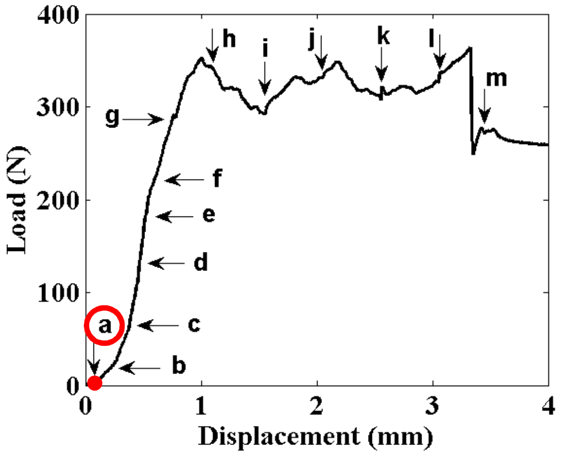
[Bouterf *et al.*, 2016, *Exp. Mech.* DOI: 10.1007/s11340-016-0168-8]

Scan Acquisitions

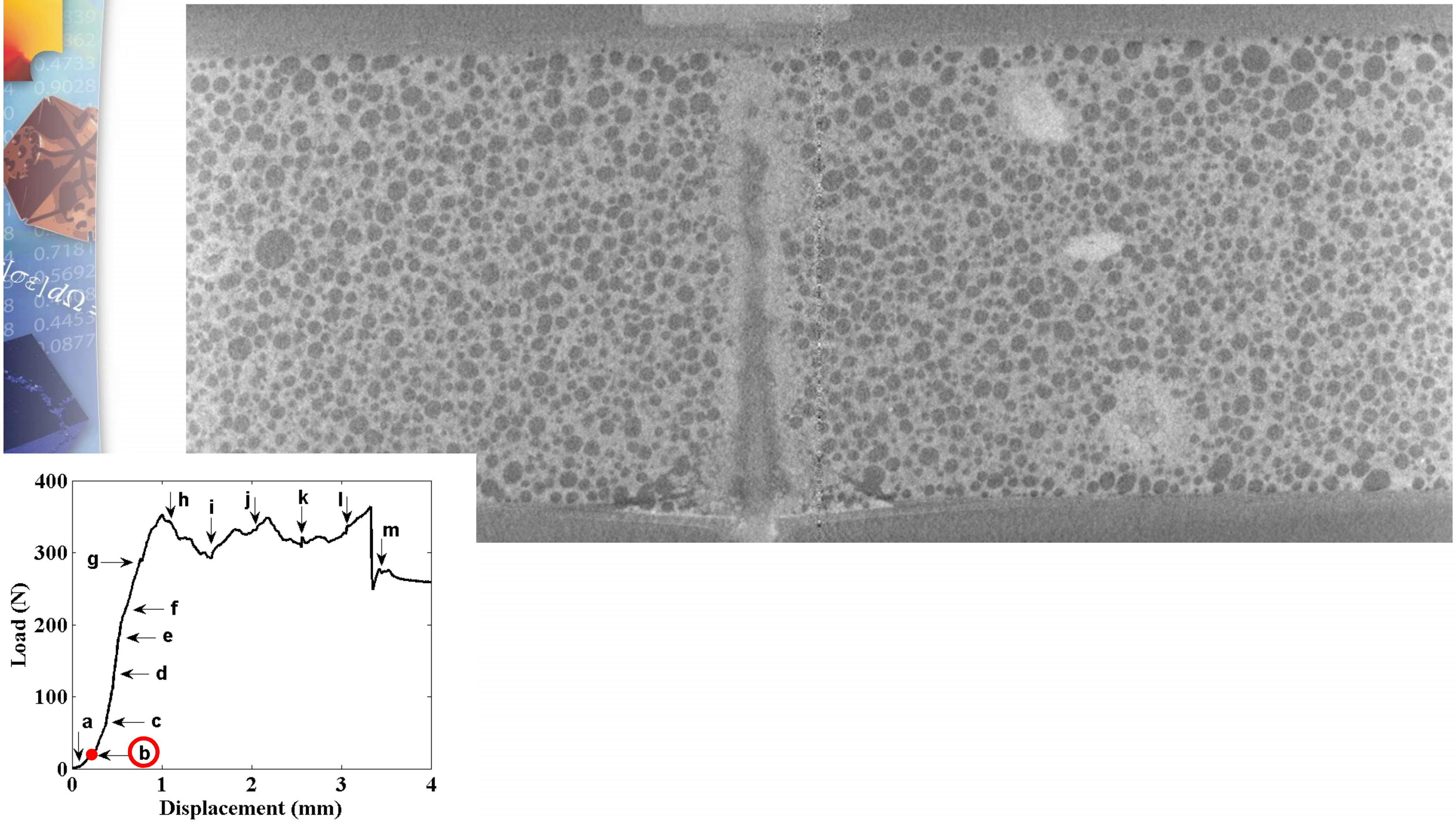


Damage Mechanisms?





Damage Mechanisms?



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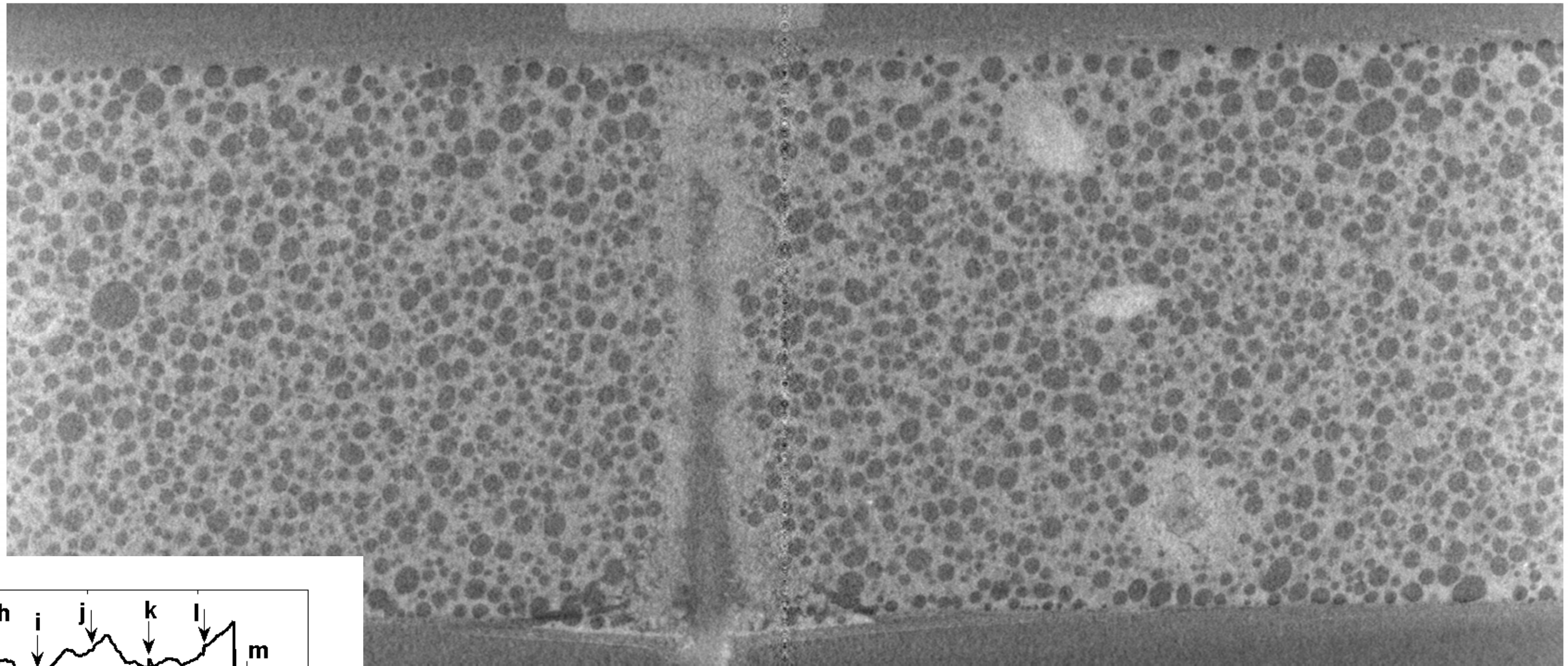

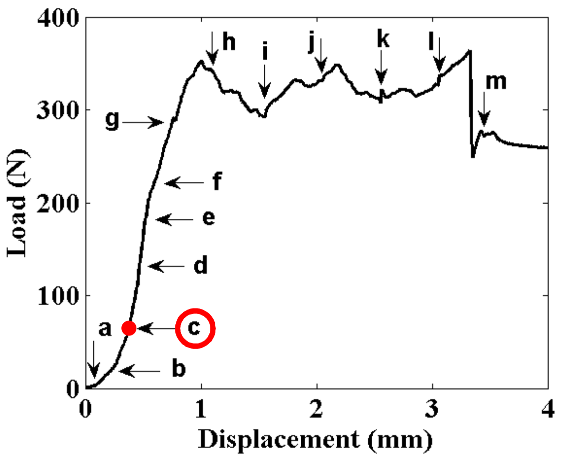

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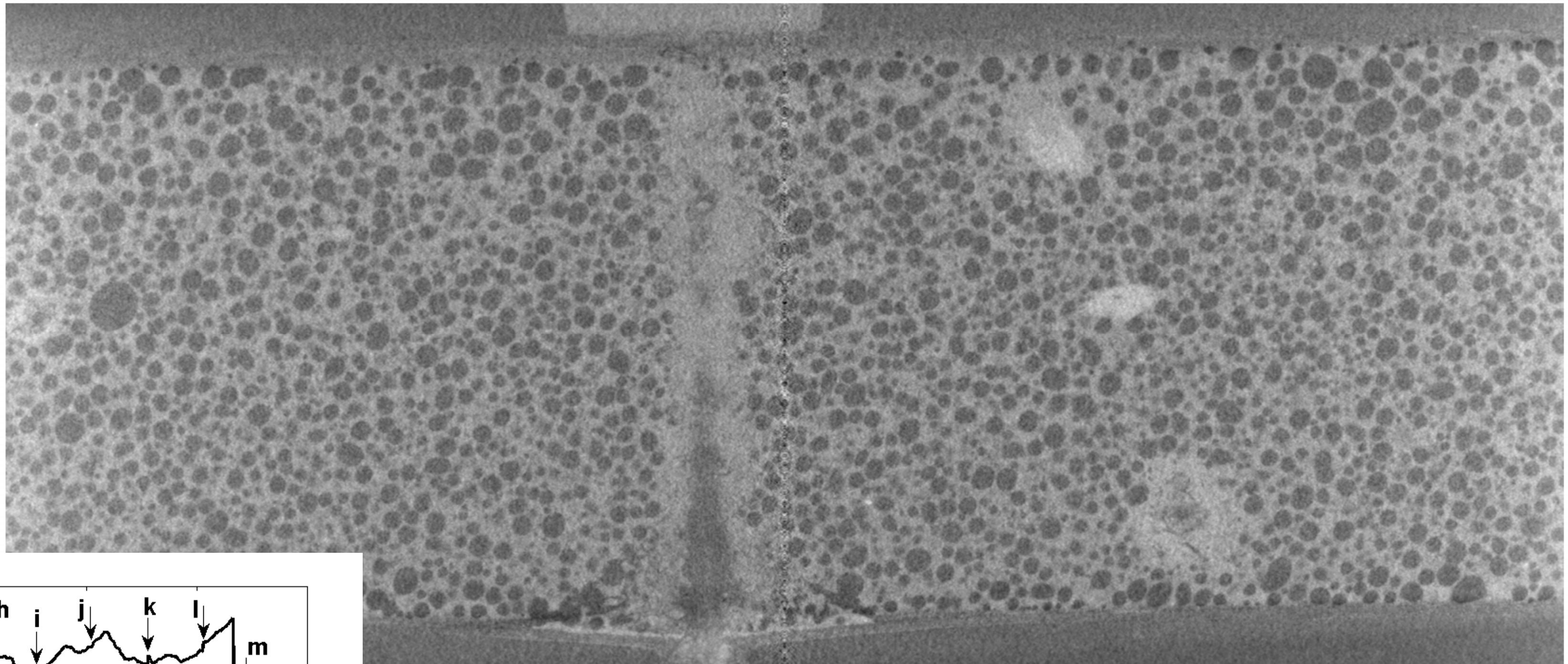

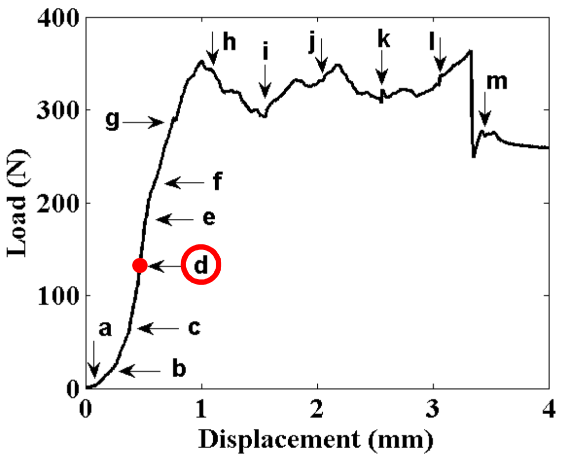

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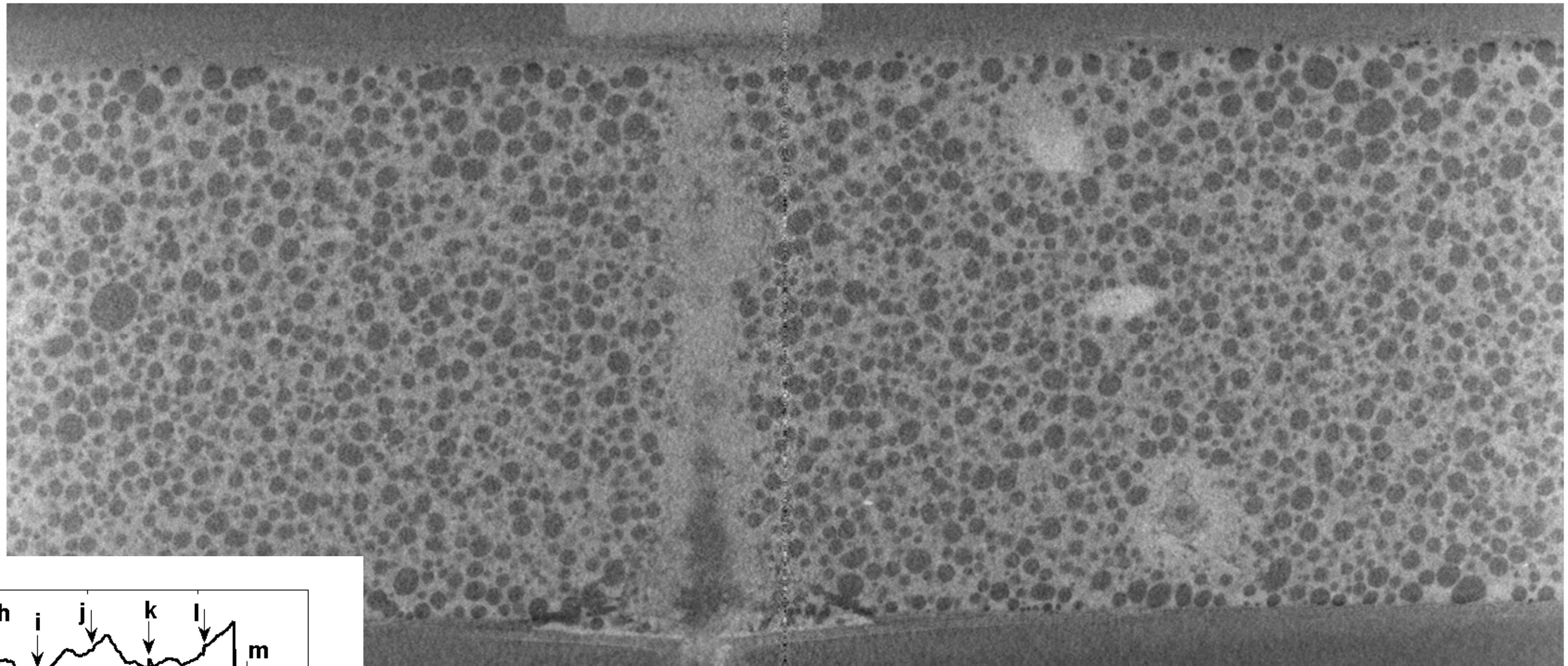

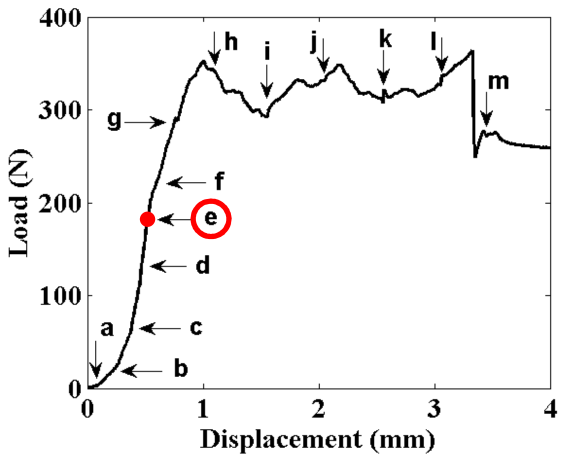

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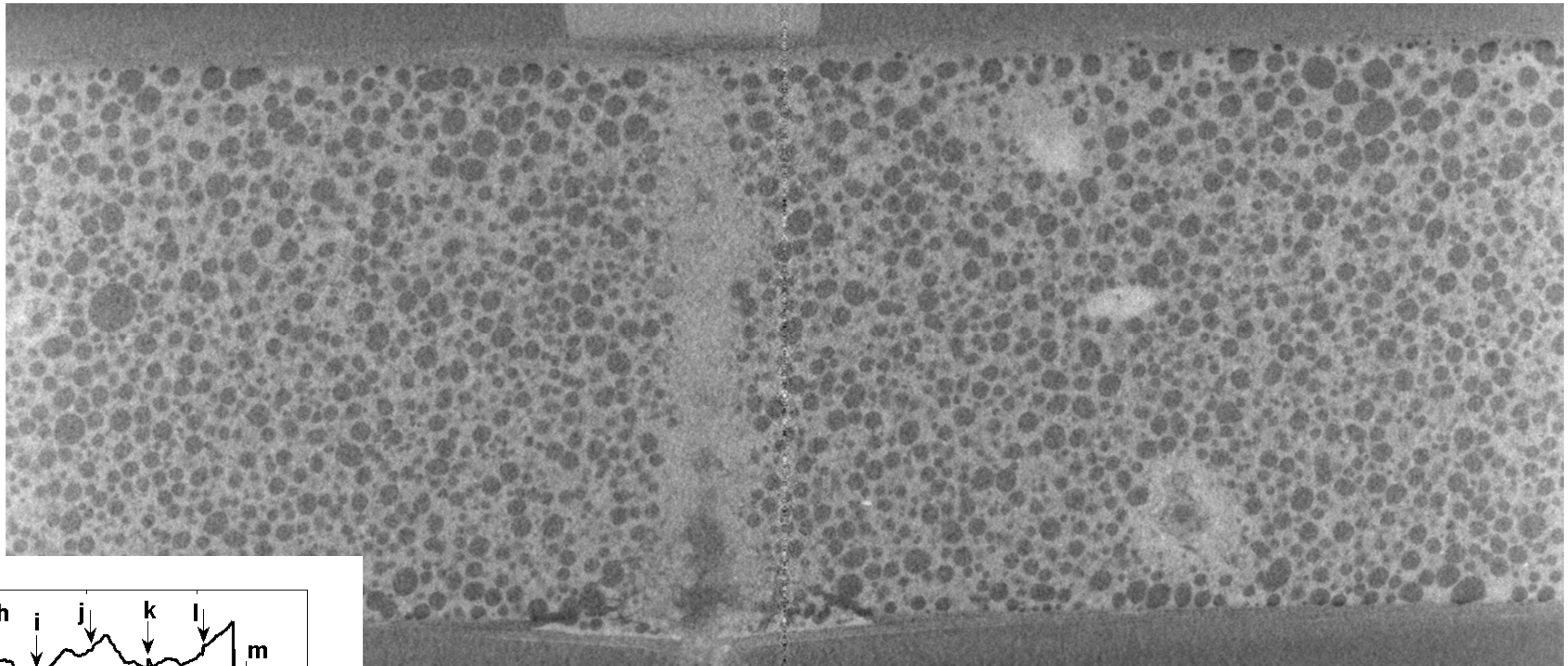

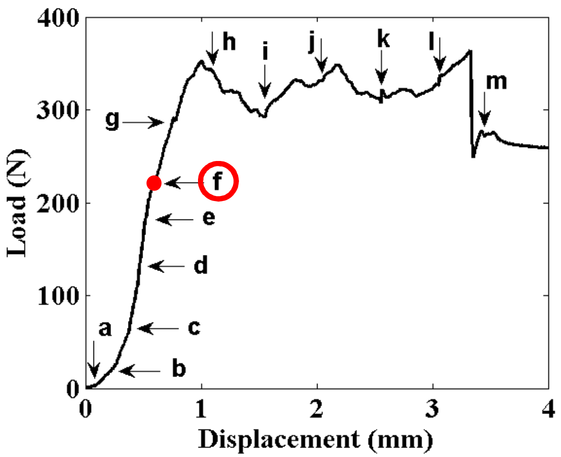

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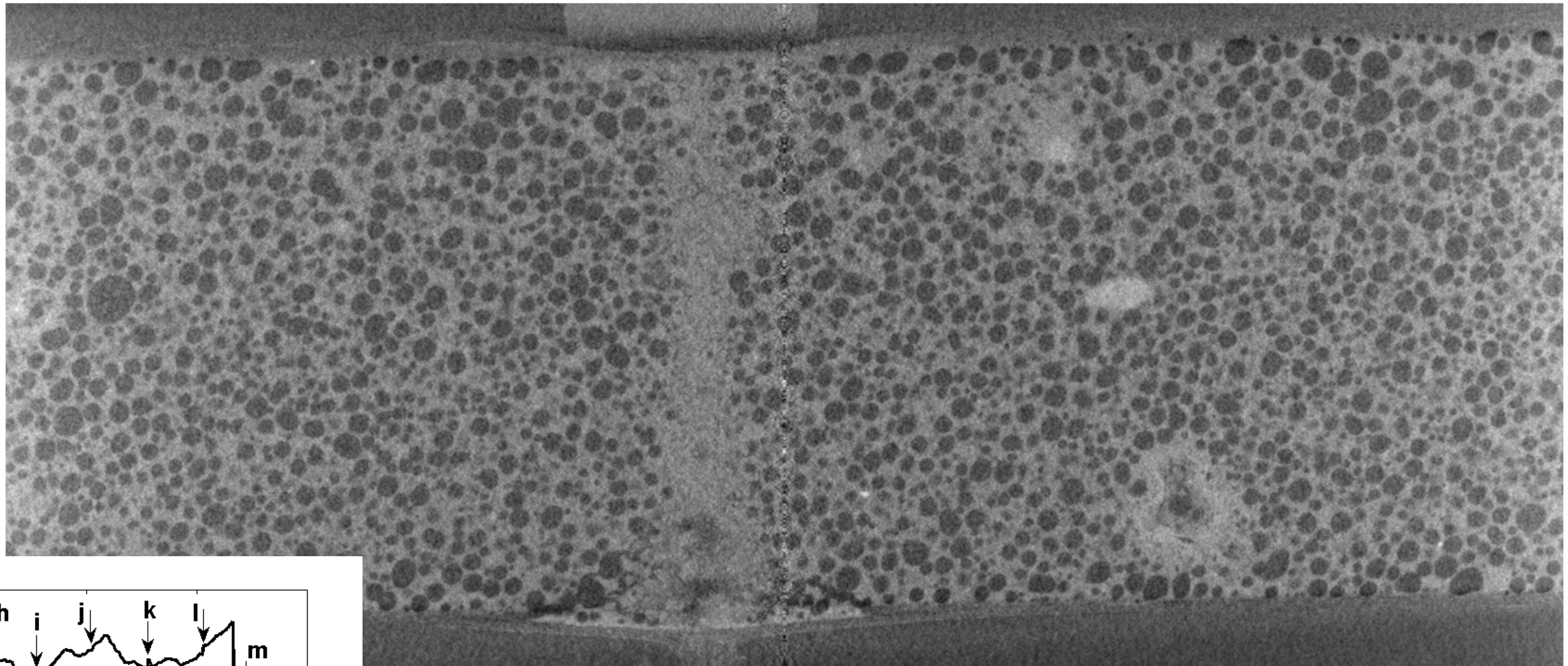

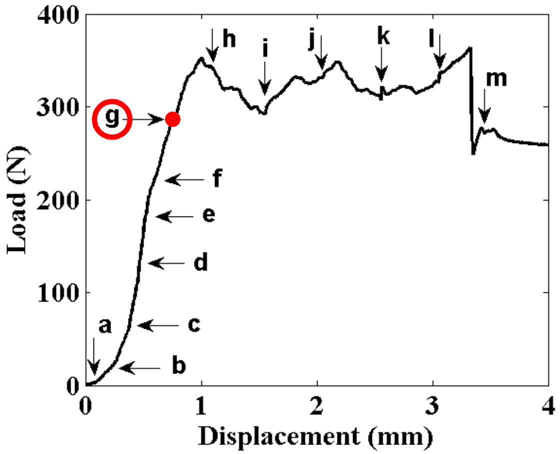

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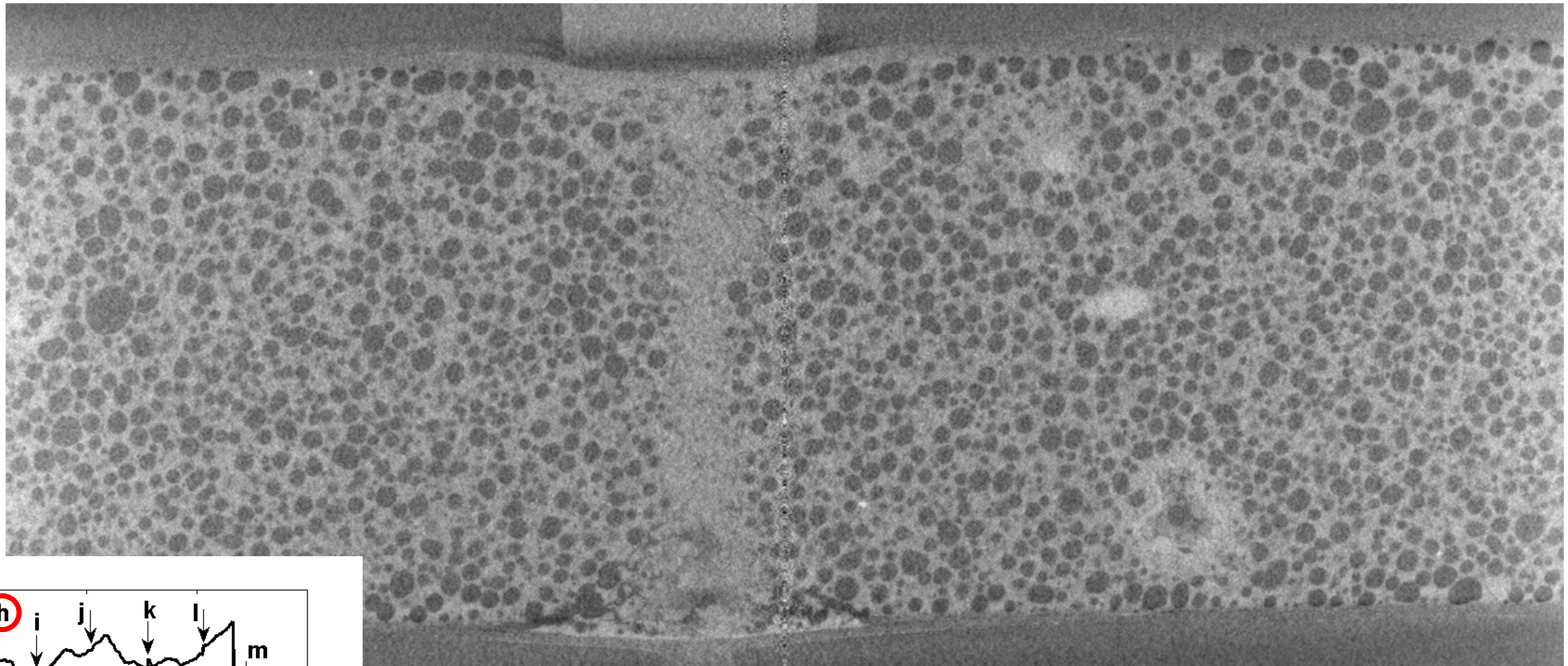

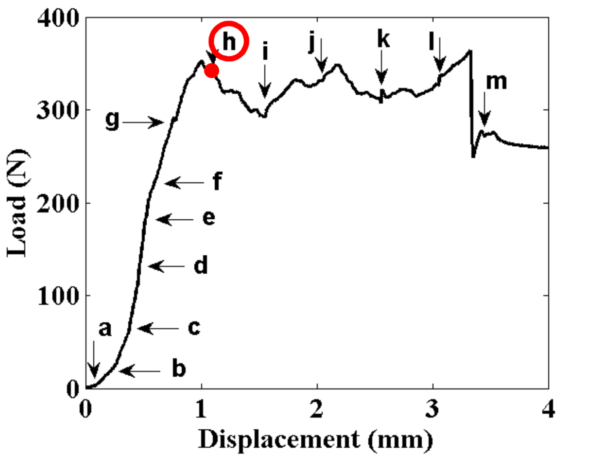

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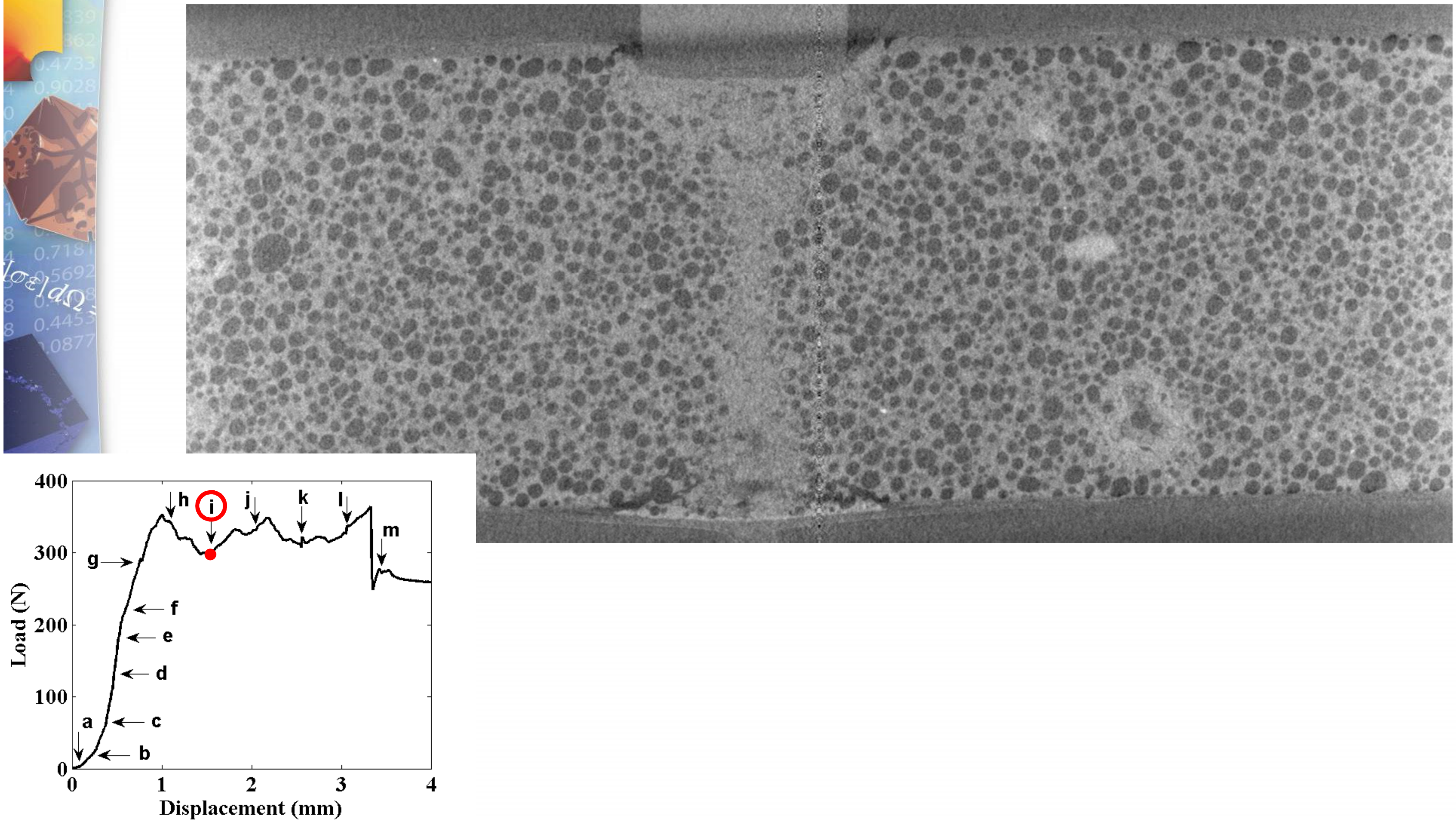

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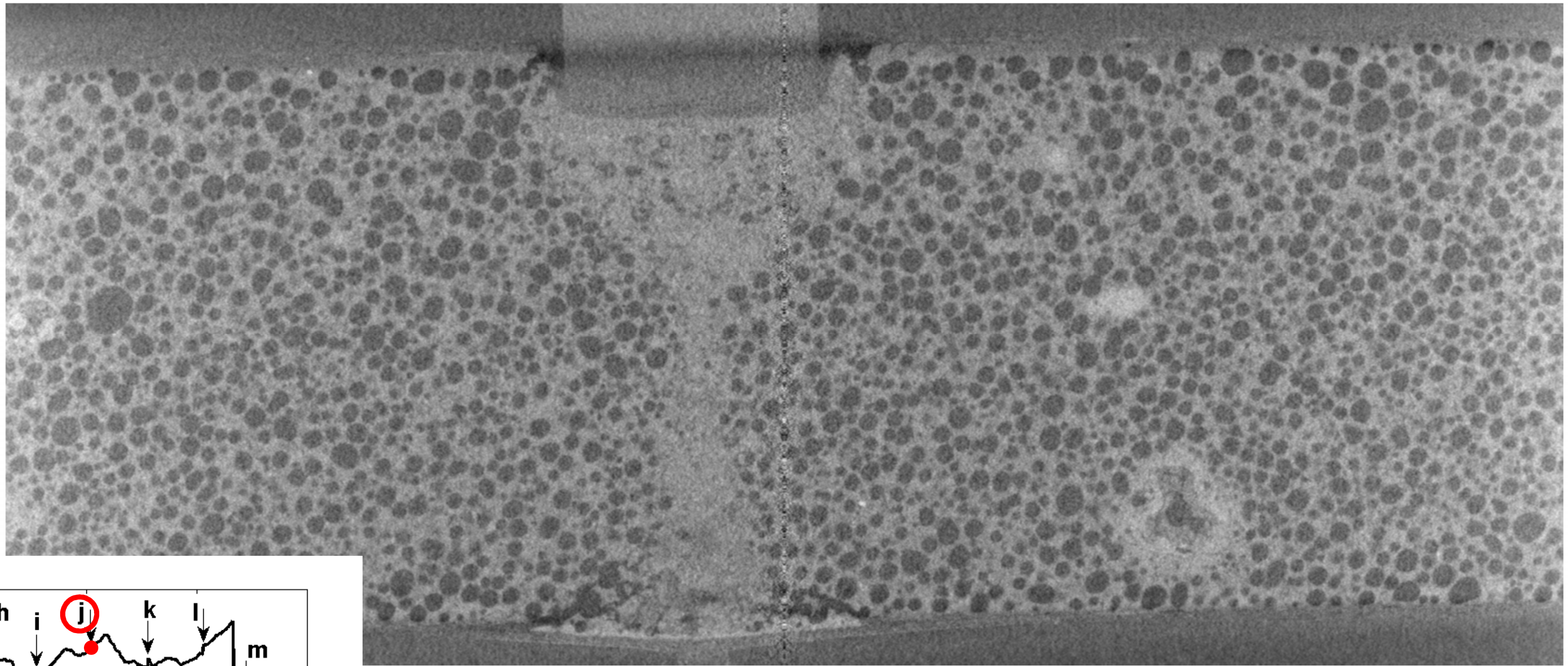

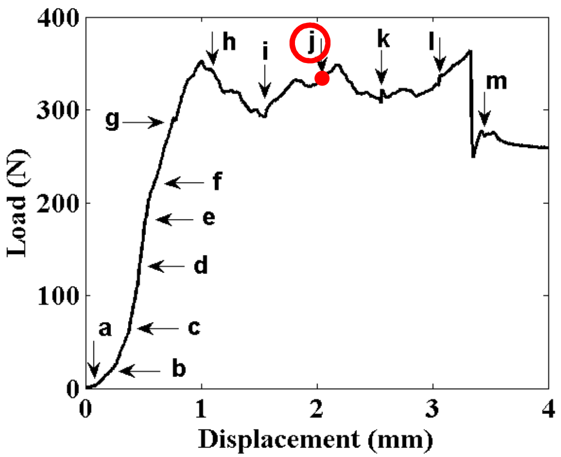

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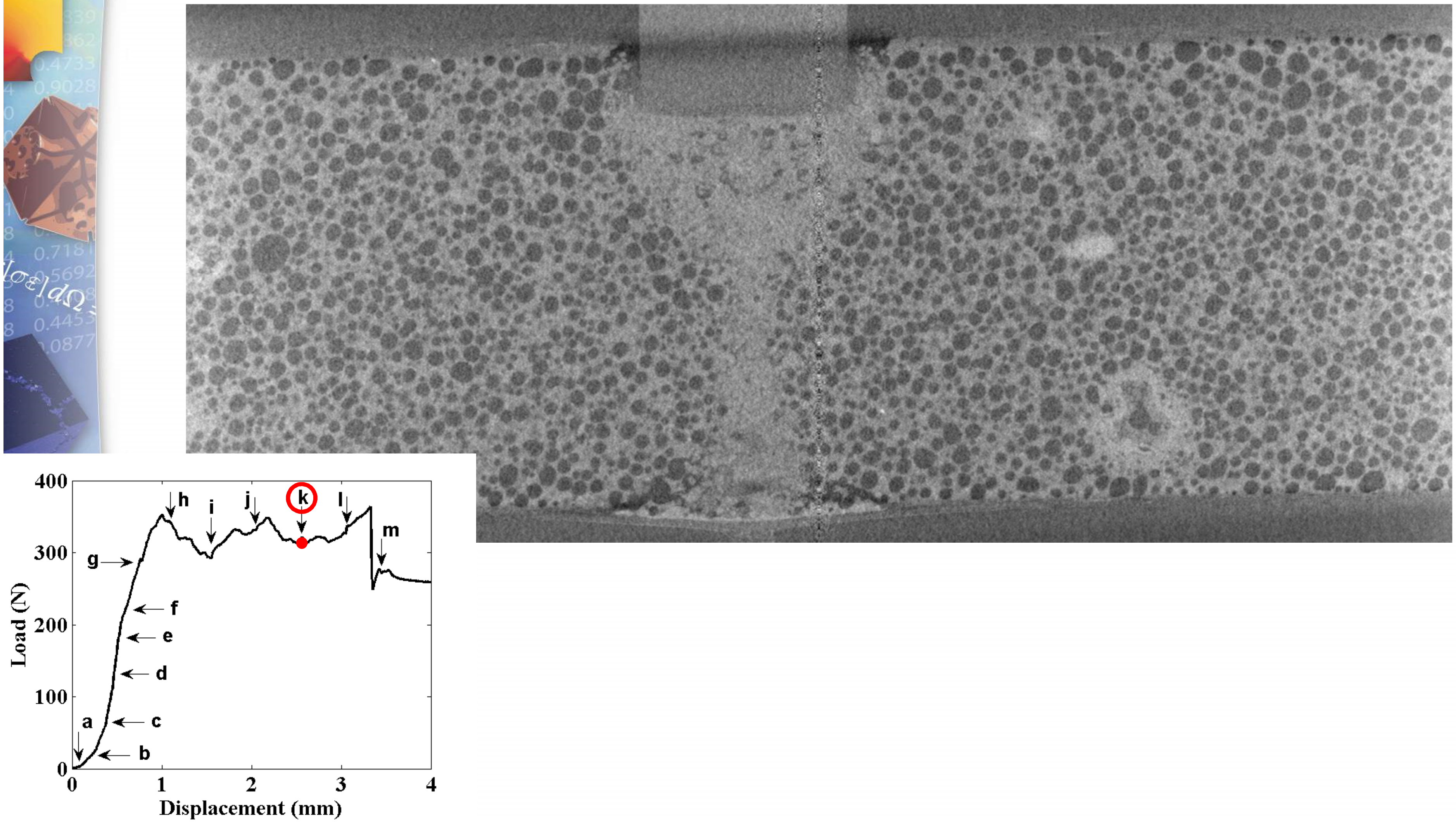

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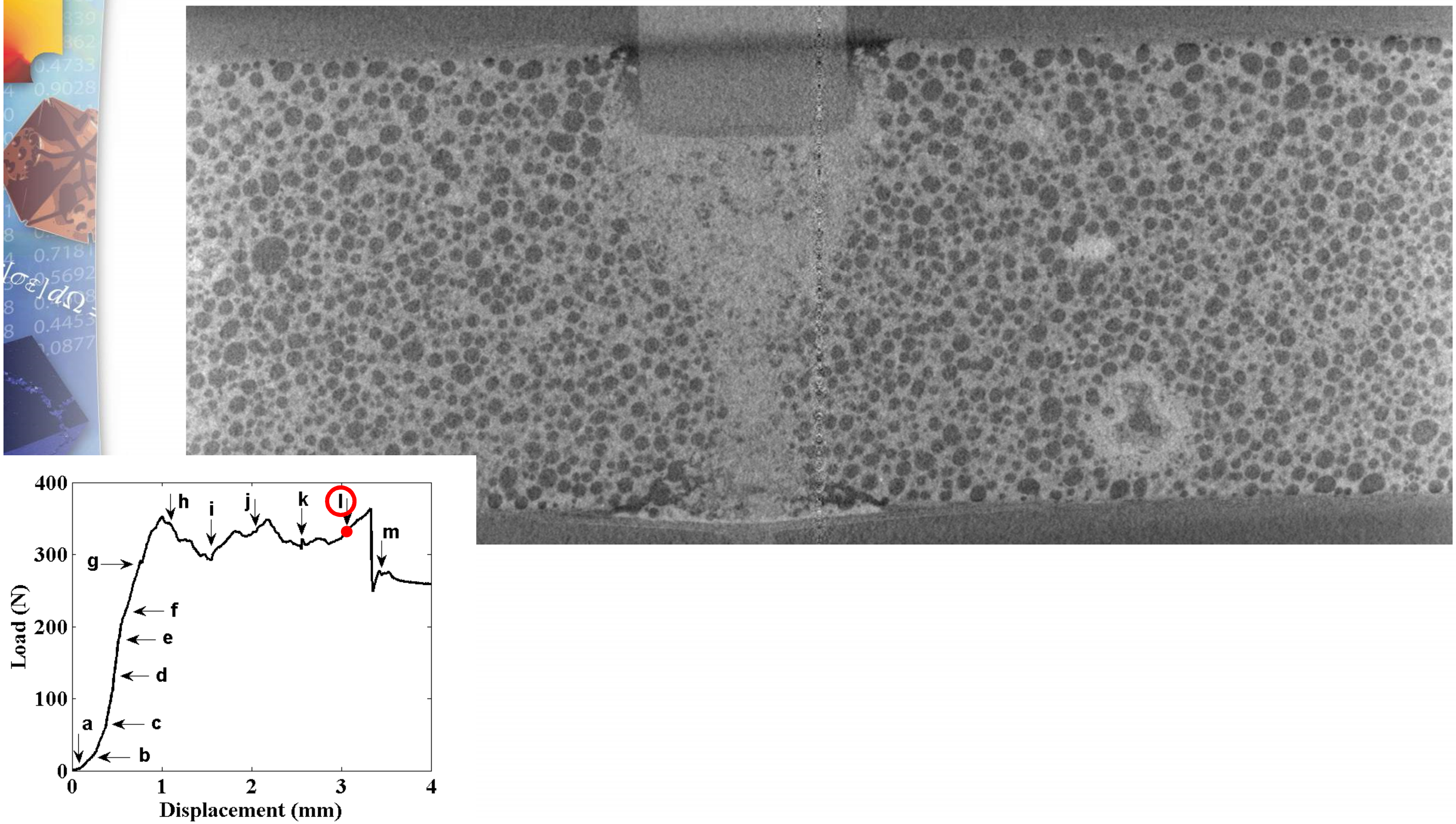

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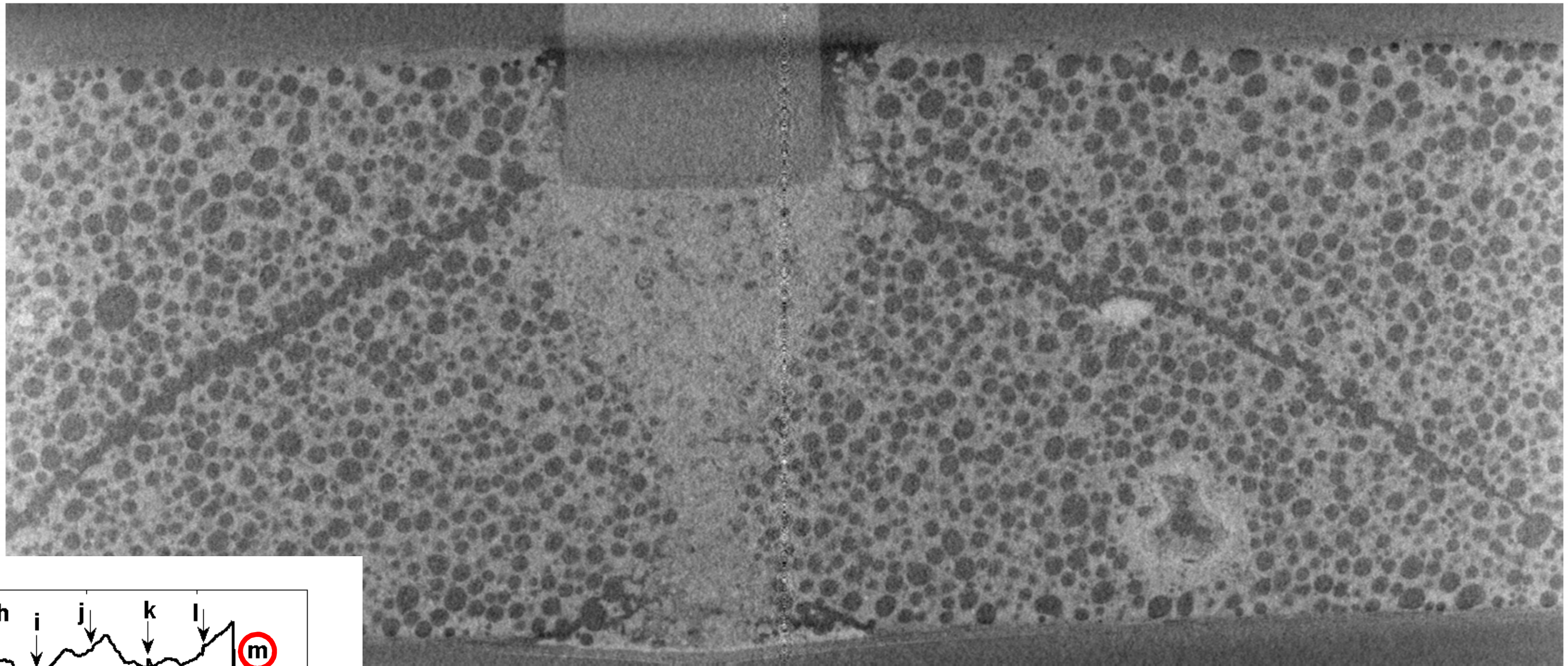
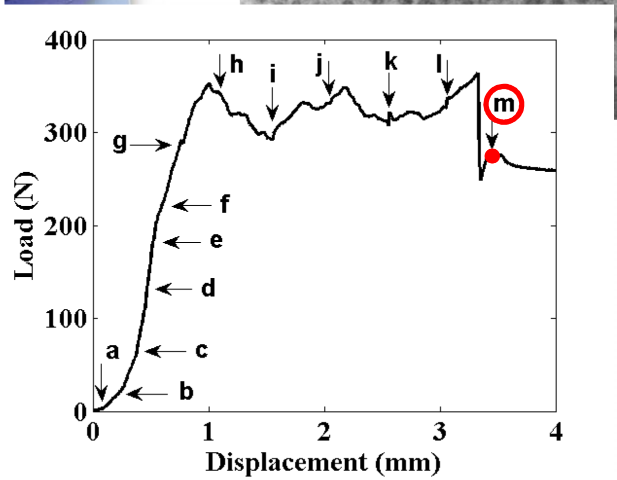
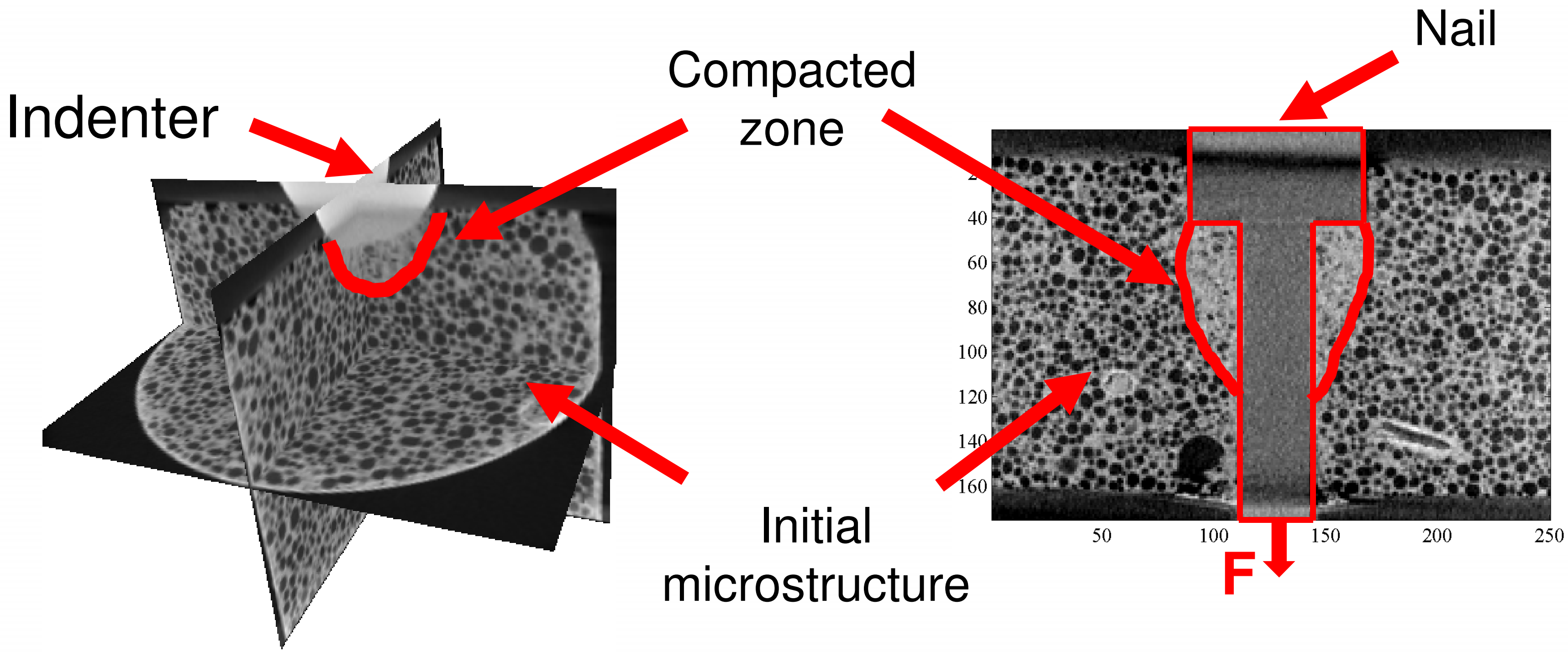




Spherical Indentation Test

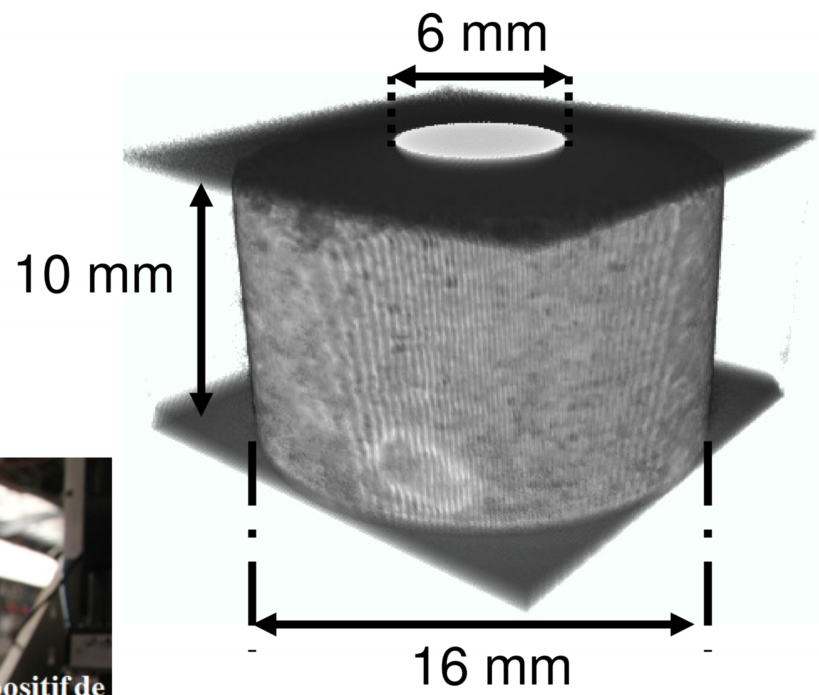
Same mechanism with simpler geometry



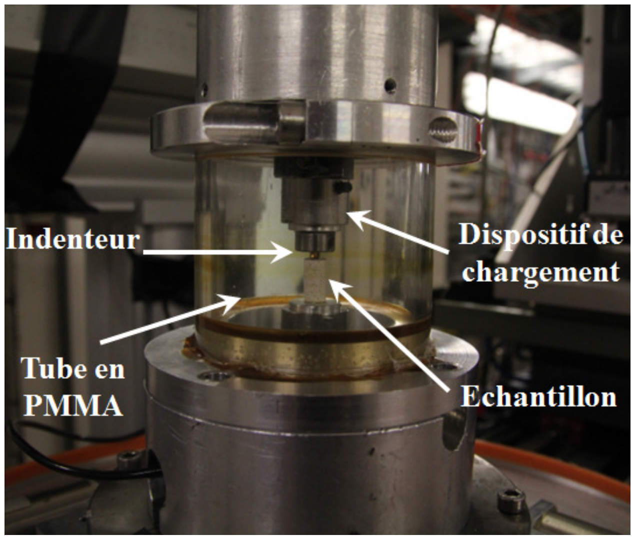
Abrupt transition
Need for strain fields at fine scale

Spherical Indentation Test

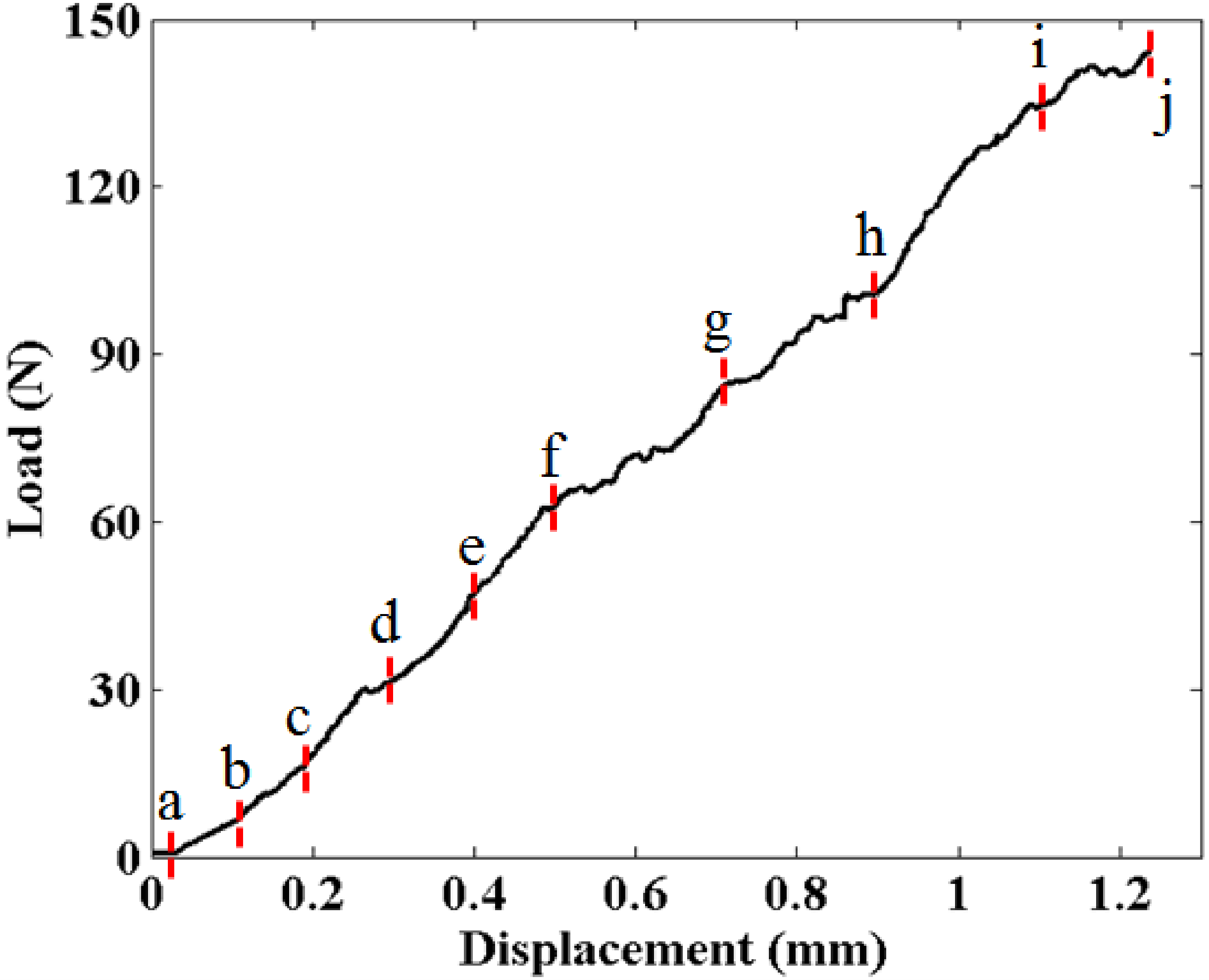
- **Aims**
  - Better understand crushing mechanism
  - Identify local failure criterion
- **Sample**



- **Identation setup**



Scan Acquisitions

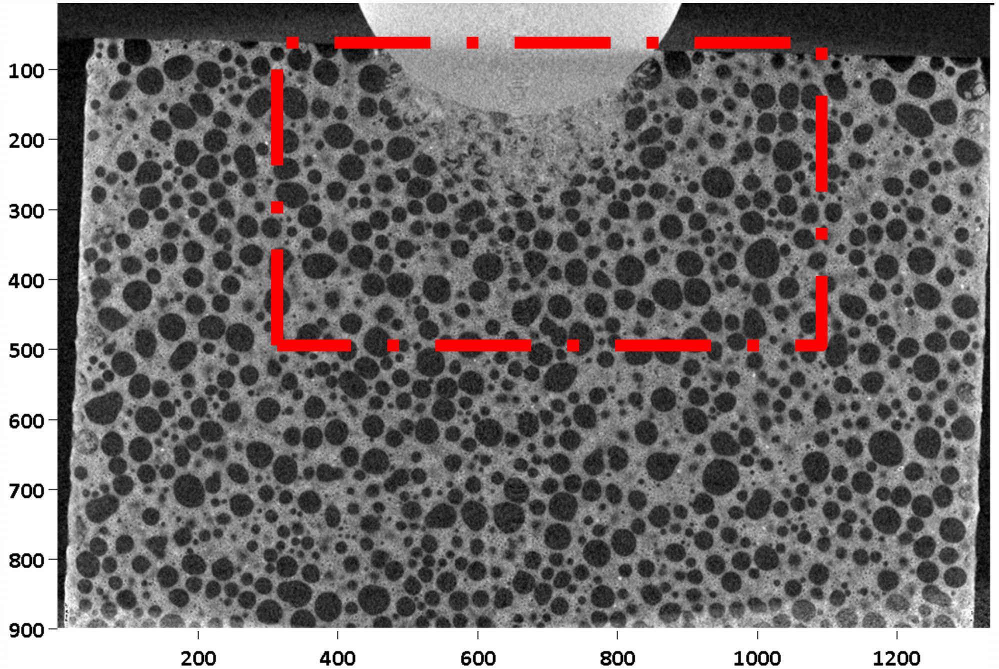


[Bouterf *et al.*, 2016, *submitted for publication*]

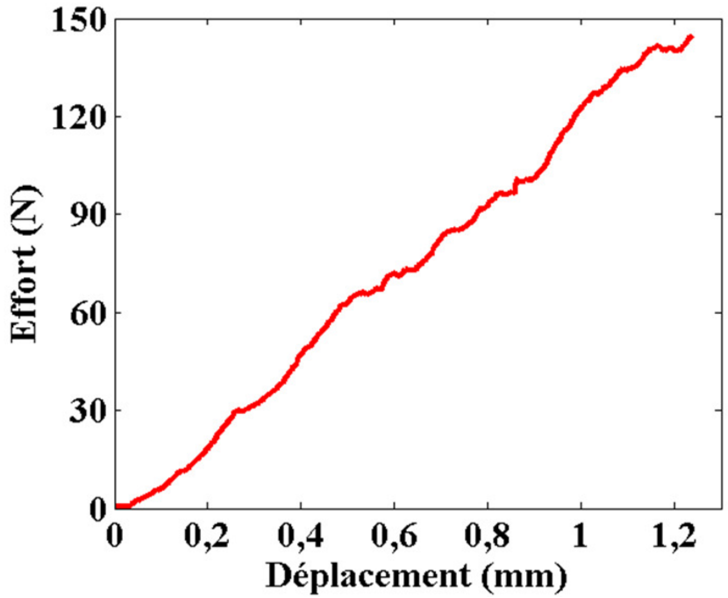
Tomographic Observations

Tomographic observations

- Compaction accompanied by pore collapse
- Same observations in artificial porous rock*



1 voxel $\leftrightarrow$ 12 µm

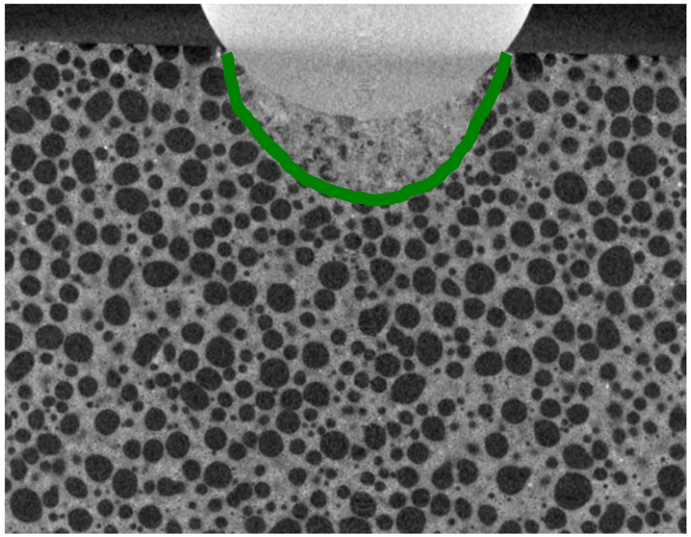


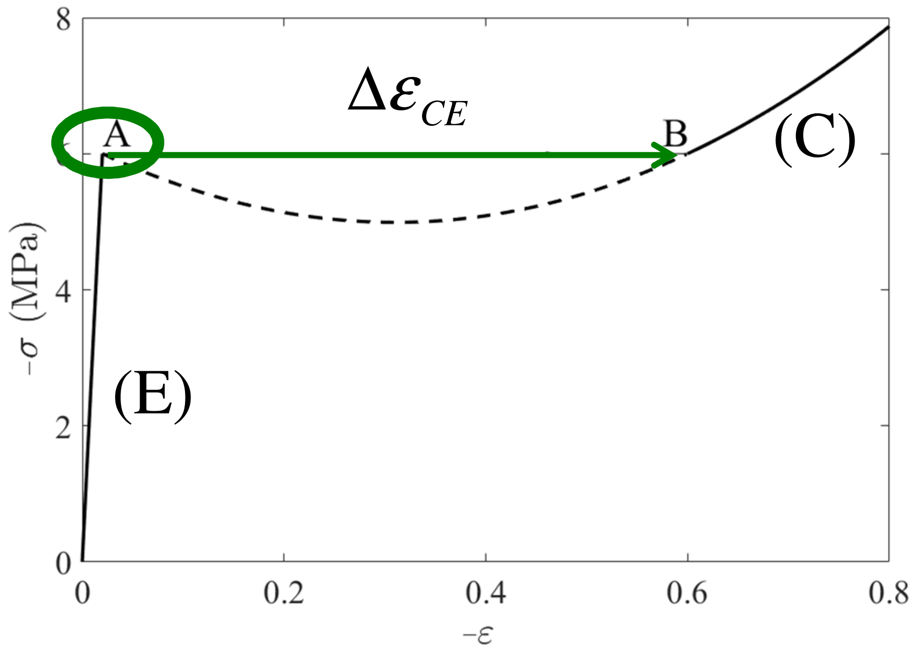
*[Leite *et al.*, 2001, *Eng. Geol.*, 59 pp. 267-280]

Spherical Indentation Test

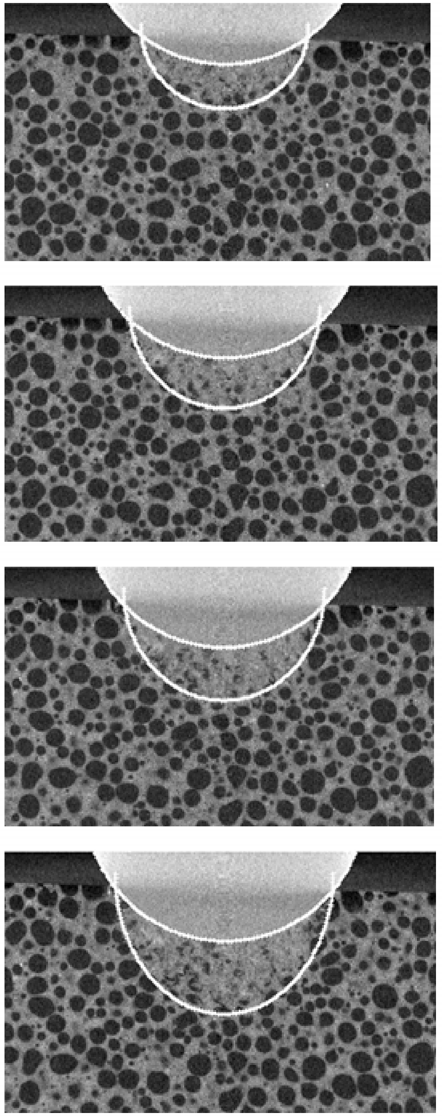
Identification of local failure criterion

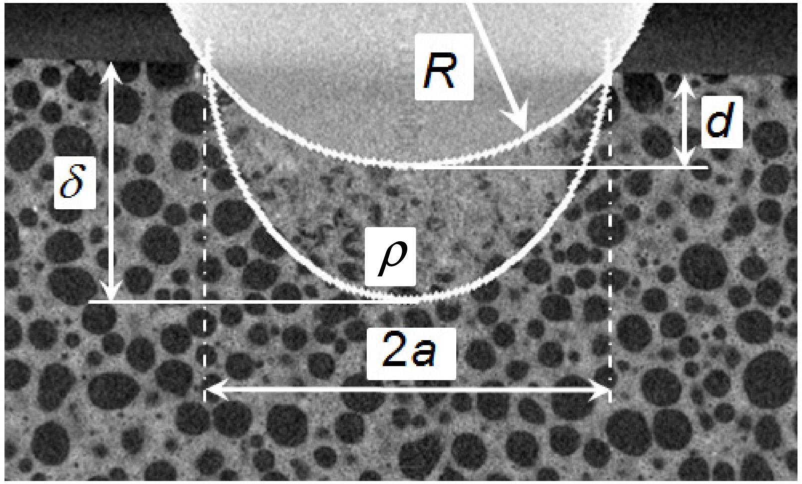
– Estimation of multiaxial stress state in transition zone





Two-Phase Model





$$a^2 = \begin{cases} 2Rd - d^2 \\ 2\rho\delta - \delta^2 \end{cases}$$

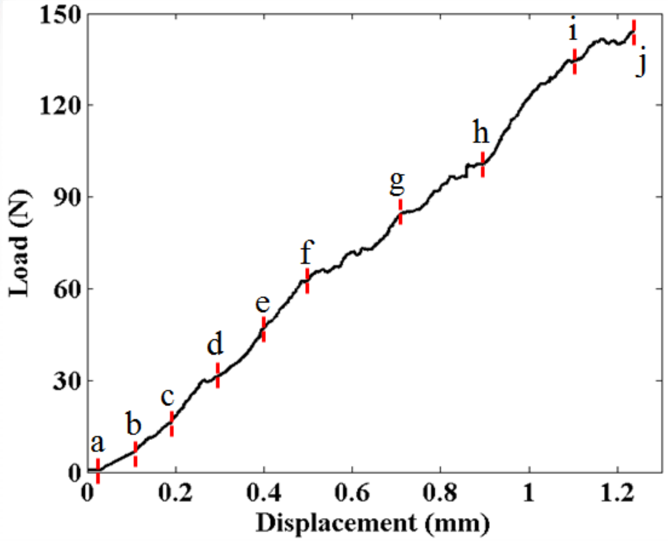
$$\phi = \frac{V_i}{V_0} = \frac{\pi R d^2 - d^3/3}{\pi\rho\delta^2 - \delta^3/3}$$

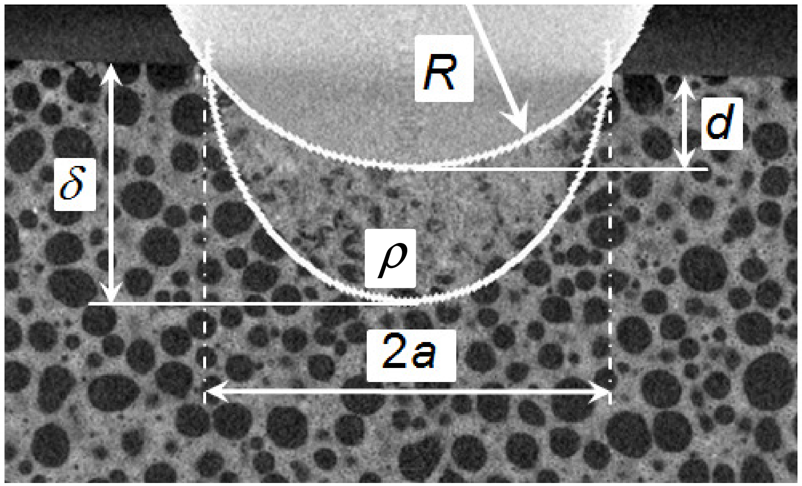
Scan	g	h	i	j
d / δ	0.44	0.47	0.50	0.46

$$\phi \approx 0.47$$

$$\Delta\varepsilon_{CE} = \log(1-\phi) \approx -0.63$$

Static Analysis





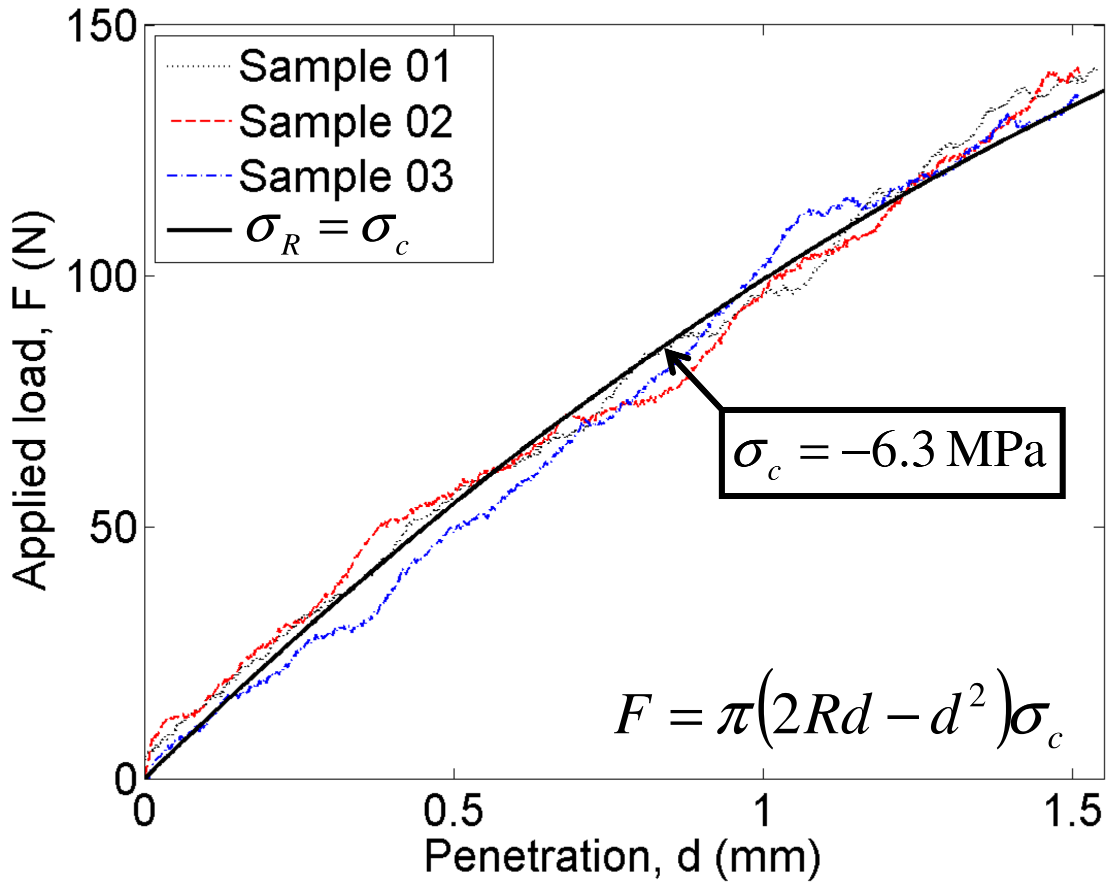
$$a^2 = \begin{cases} 2Rd - d^2 \\ 2\rho\delta - \delta^2 \end{cases}$$

$$\sigma_{eff} = \frac{F}{\pi a^2} = \frac{F}{\pi\left(2Rd - d^2\right)} = \sigma_c$$

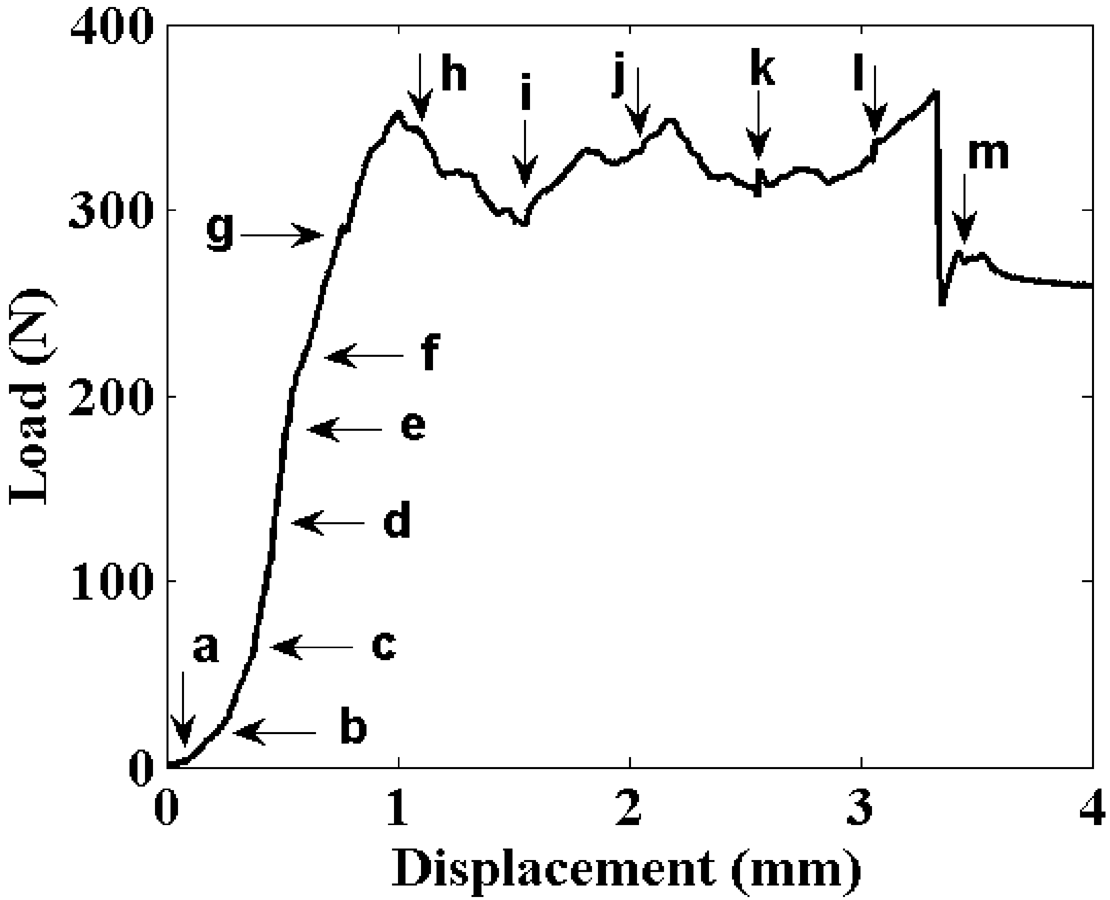
Static Analysis

No shear flow (surface normal = eigen stress direction)

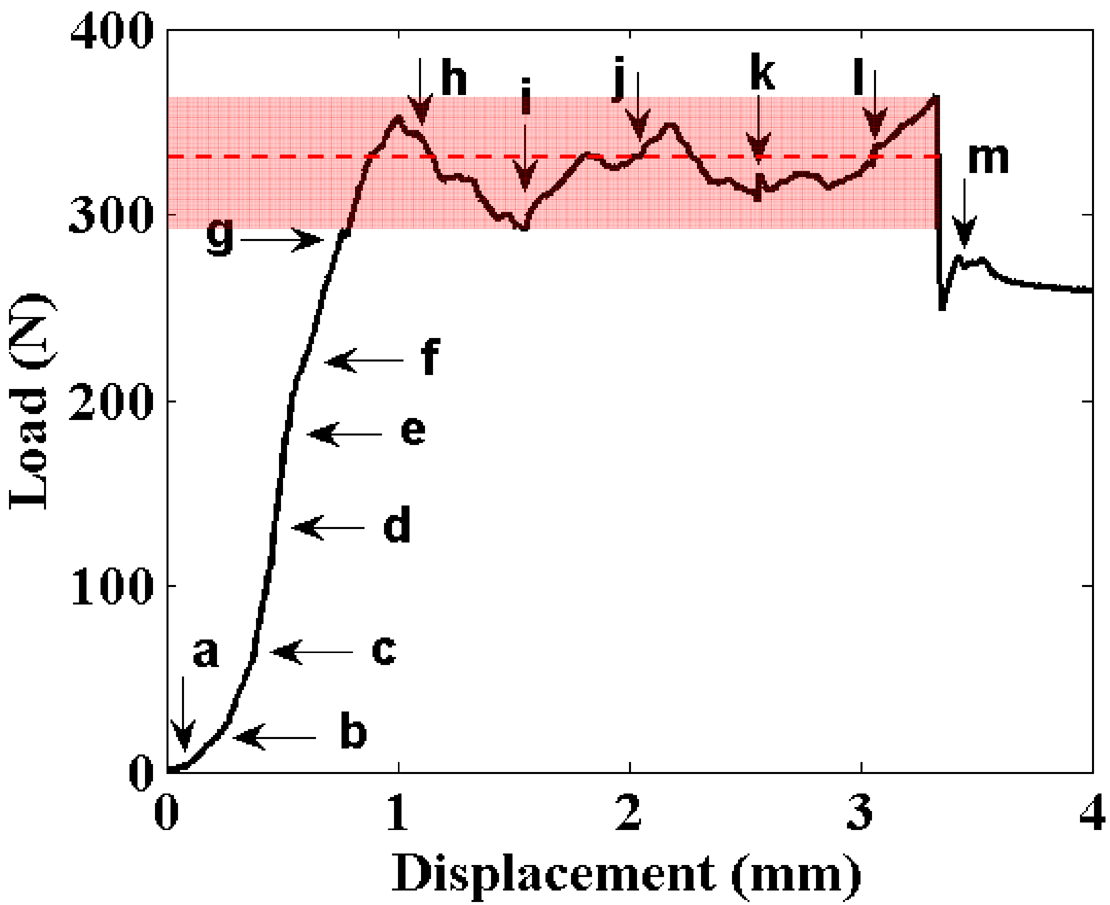
$$F_z = \pi a^2 \sigma_R$$



Nail Pull Test



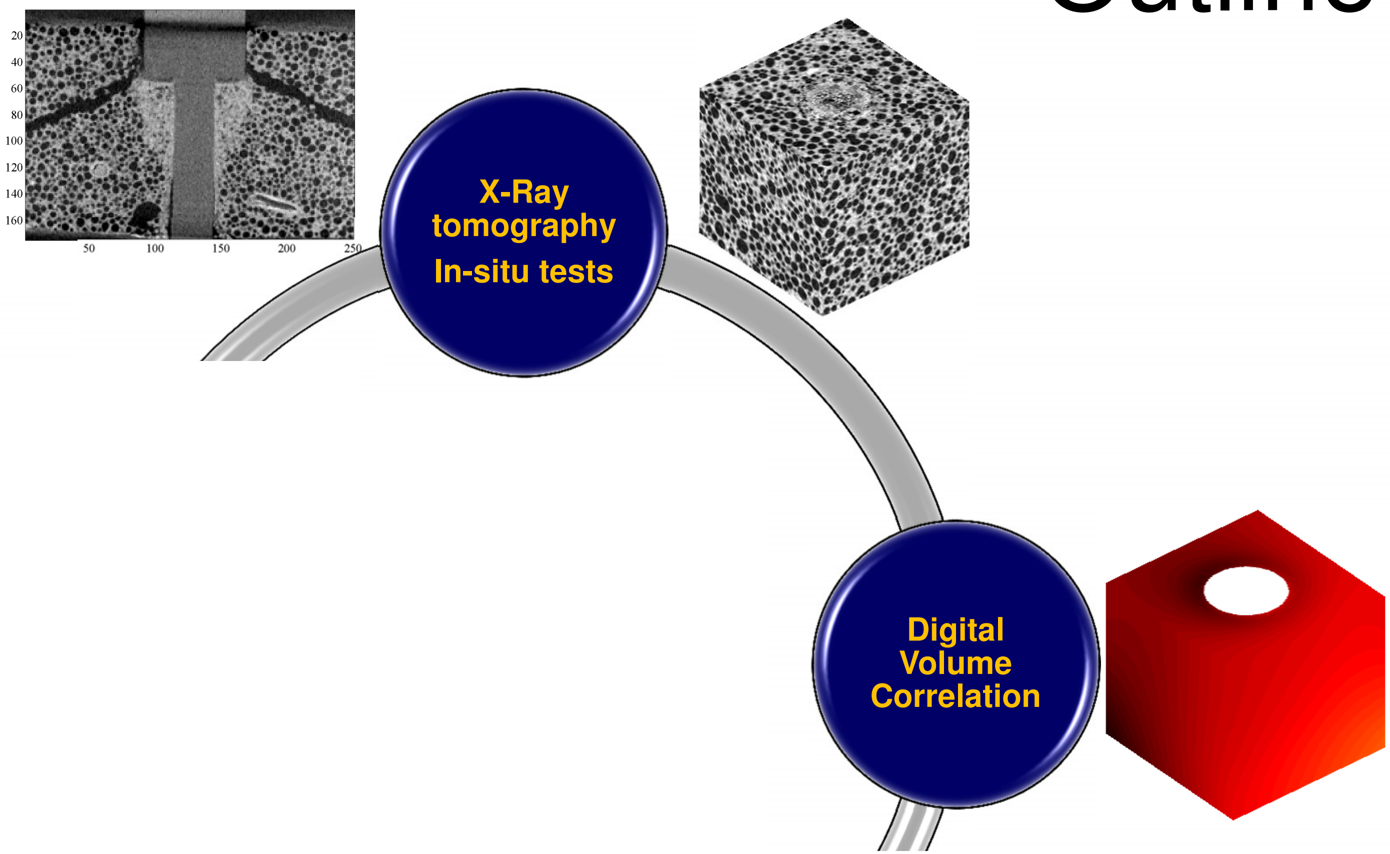
Nail Pull Test



$$F = 320 \pm 25\ \mathrm{N} \rightarrow \sigma_c = -6.3 \pm 0.5\ \mathrm{MPa}$$

[Bouterf *et al.*, 2016, *Exp. Mech.* DOI: 10.1007/s11340-016-0168-8]

Outline



X-Ray tomography
In-situ tests
Digital Volume Correlation
20
40
60
80
100
120
140
160
50
100
150
200
250

Digital Volume Correlation (DVC)

- Gray level (reconstructed) volumes

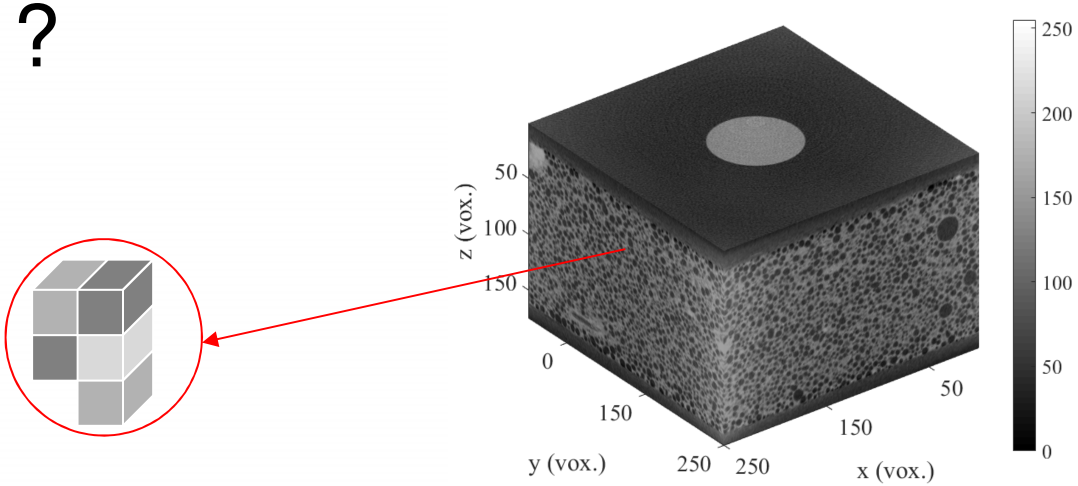
$$f(\underline{x}) \qquad g(\underline{x})$$

- Conservation of gray levels

$$f(\underline{x}) \cong g(\underline{x} + \underline{u}(\underline{x}))$$

- Measure $\underline{u}(\underline{x})$?

voxels



Digital Volume Correlation

- Local registration:
  - Biomechanics
    [Bay *et al.*, 1999; 2002; Verhulp *et al.*, 2004; Tong *et al.*, 2009…]
  - Mechanics of materials
    [Bornert *et al.*, 2004; Franck *et al.*, 2007; Germaneau *et al.*, 2007; Lenoir *et al.*, 2007; Forsberg *et al.*, 2008…]

- Global registration:
  - Biomechanics
    [Benoit *et al.*, 2009; Madi *et al.*, 2013]
  - Mechanics of materials
    [Roux *et al.*, 2008; Réthoré *et al.*, 2008; HF *et al.*, 2009; Limodin *et al.*, 2009…]

Global Approach to DVC

- Select a specific displacement basis $\underline{\varphi}_i(\underline{x})$ such that

$$\underline{u}(\underline{x}) = \sum_i a_i \underline{\varphi}_i(\underline{x})$$

- Minimize correlation residuals*

$$\rho_c^2(\{a_i\}) = \iiint \left[f(\underline{x}) - g(\underline{x} + a_i \underline{\varphi}_i(\underline{x})) \right]^2 d\underline{x}$$

- Successive linearizations / corrections

$$M_{ij} \delta a_j = b_i$$

*[Roux *et al.*, 2008, *Comp. Part A* 39 pp. 1253-1265]

Finite Element DVC

$$\rho^2_{\mathrm{lin}}(\delta\underline{u}) = \int_{\Omega} \left[f(\underline{x}) - \hat{g}(\underline{x}) - \left(\delta\underline{u} \cdot \underline{\nabla} f\right)(\underline{x}) \right]^2 d\underline{x}$$

$$= \sum_{e} \int_{\Omega_e} \left[f(\underline{x}) - \hat{g}(\underline{x}) - \delta a_i^e \left(\underline{\varphi}_i \cdot \underline{\nabla} f\right)(\underline{x}) \right]^2 d\underline{x}$$

Elementary matrix and vector (e.g., C8P1*)

$$M_{ij}^e = \int_{\Omega_e} \left(\underline{\nabla} f \cdot \underline{\varphi}_i\right)(\underline{x}) \left(\underline{\nabla} f \cdot \underline{\varphi}_j\right)(\underline{x})\, d\underline{x}$$

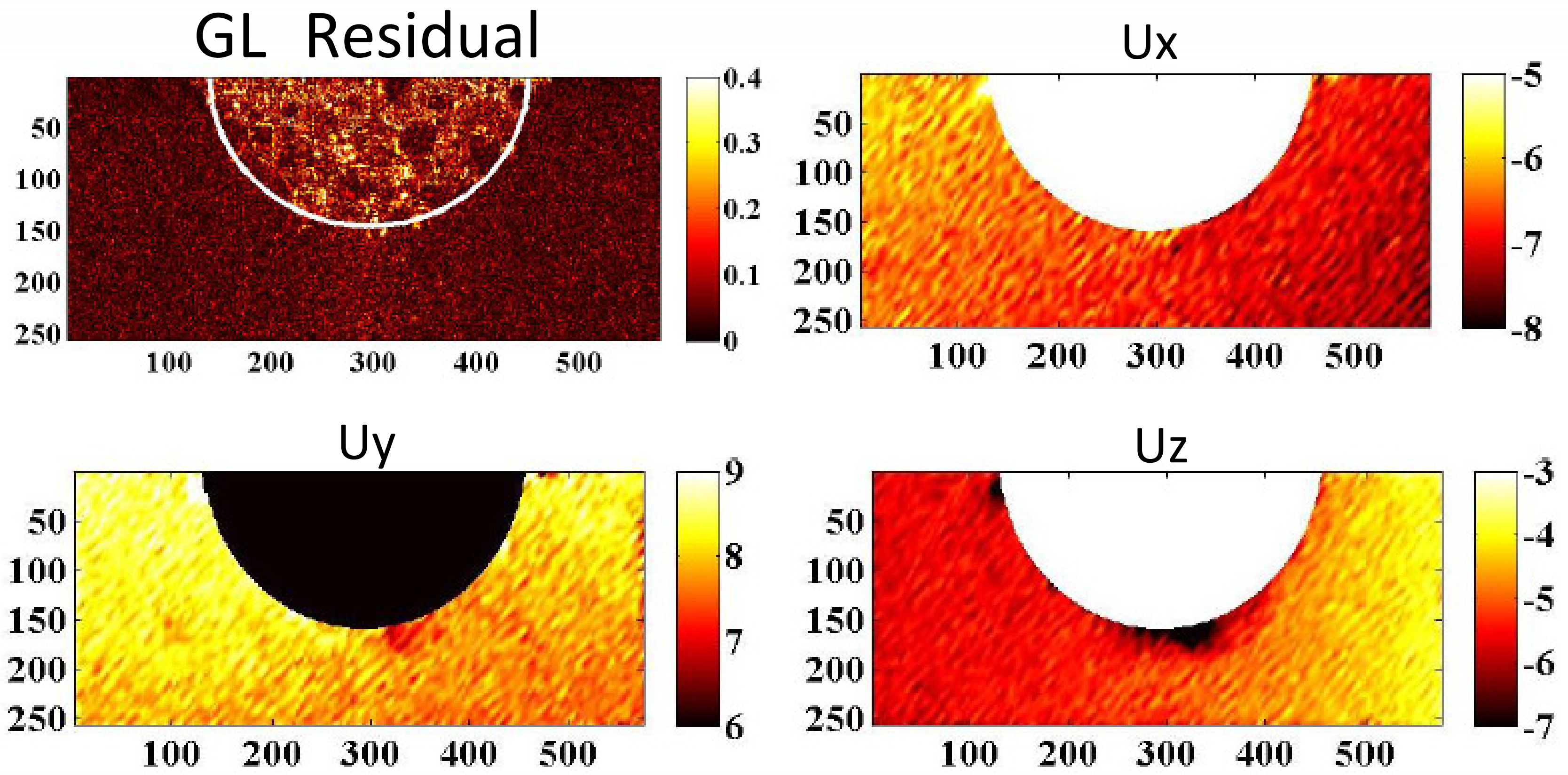
$$b_i^e = \int_{\Omega_e} \left[f(\underline{x}) - \hat{g}(\underline{x}) \right] \left(\underline{\nabla} f \cdot \underline{\varphi}_i\right)(\underline{x})\, d\underline{x}$$

*[Roux *et al.,* 2008, *Comp. Part A* 39 pp. 1253-1265]

C8-DVC Analyses

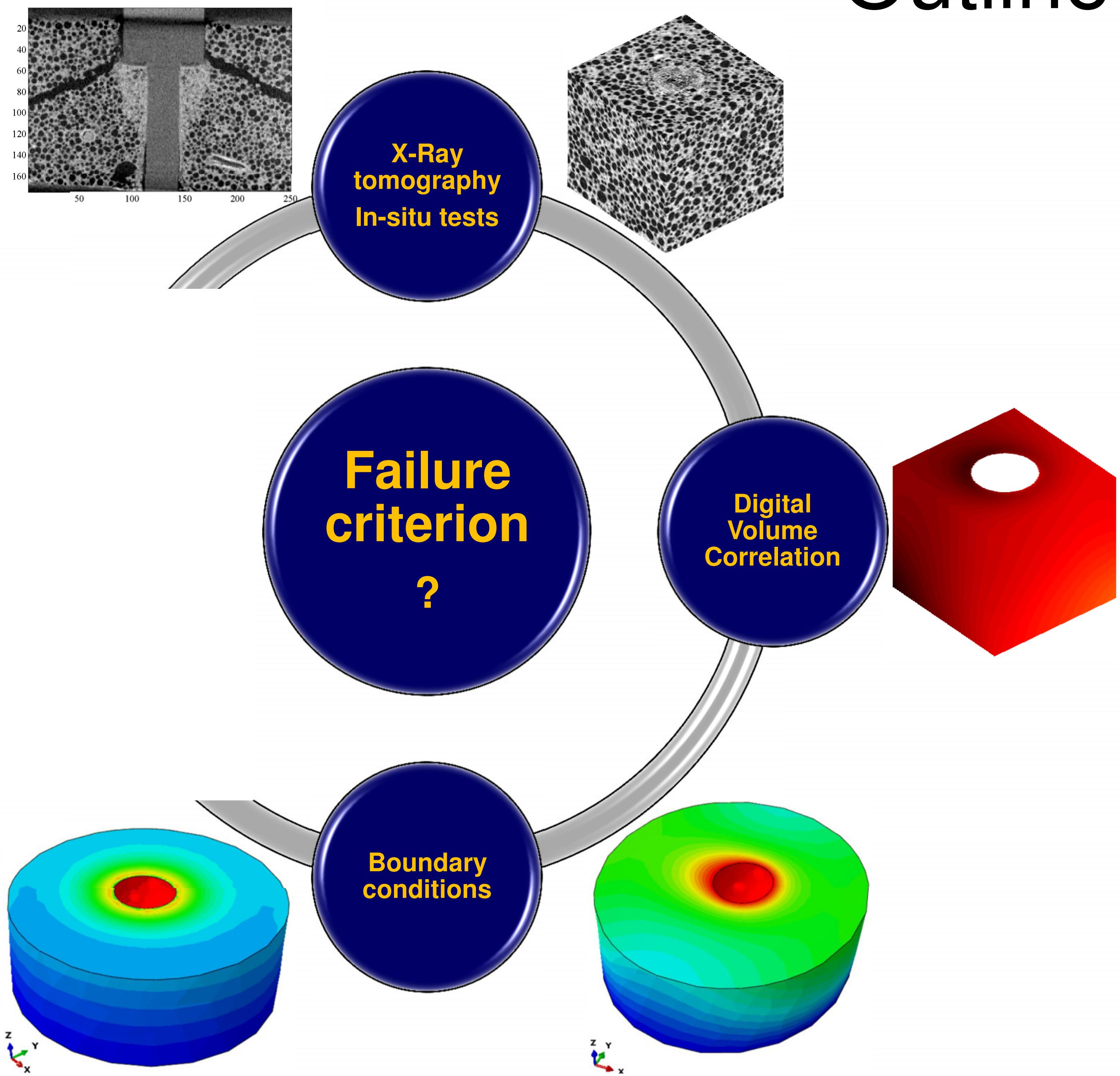
Small amplitude displacements

- Standard displacement resolution ≈ 0.4 voxel (ℓ = 8 voxels)
- Unreliable elastic strain estimates



1 voxel ↔ 12µm

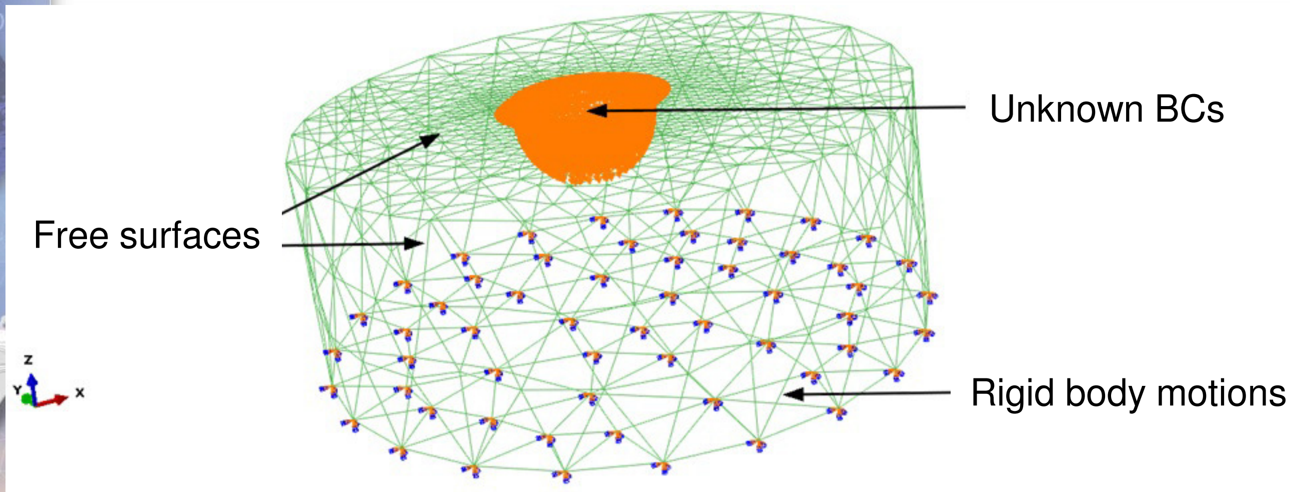
Outline

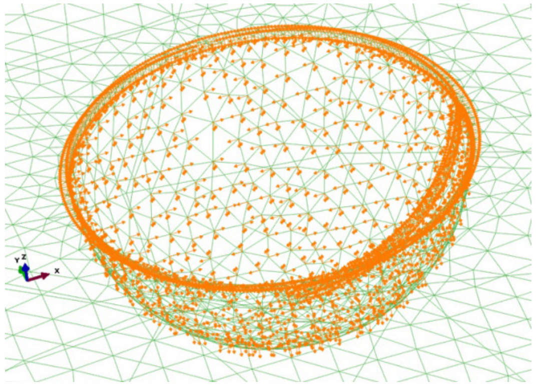

X-Ray tomography
In-situ tests
Failure criterion
?
Digital Volume Correlation
Boundary conditions

Reduced Kinematic Basis

FE-generated kinematic basis

- Isotropic elasticity
- Crushed zone excluded
- T4 mesh
- Dirichlet boundary conditions
  - measured rigid body motion (C8-DVC)
  - linear combination of modes





[Bouterf *et al.*, 2014, *Strain* 50 pp. 444-453]

Reduced Kinematic Basis

- **BCs under crushed zone**
  - **Axisymmetric fields**

$$u_x(a_i,\varphi,\theta) = v\cos(\varphi)\sin(\theta) + w\cos(\varphi)\cos(\theta)$$

$$u_y(a_i,\varphi,\theta) = v\sin(\varphi)\sin(\theta) + w\sin(\varphi)\cos(\theta)$$

$$u_z(a_i,\varphi,\theta) = v\cos(\varphi) - w\sin(\theta)$$

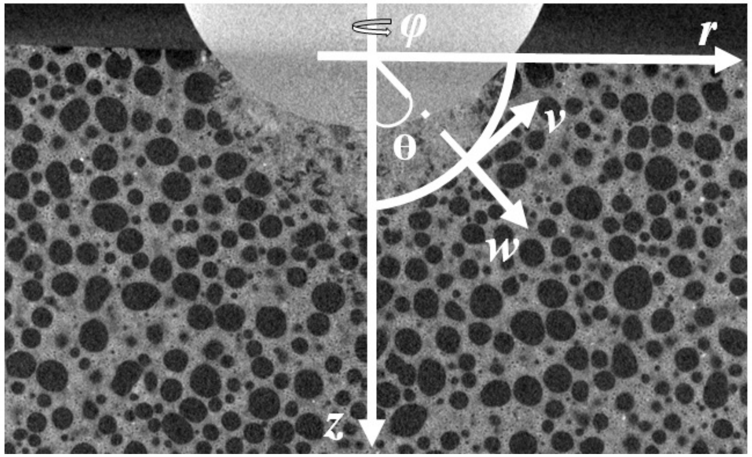
θ and φ are polar et azimuthal angles

$$v = a_1 + a_2\cos(\theta) + a_3\cos(2\theta) + a_4\cos(3\theta)$$

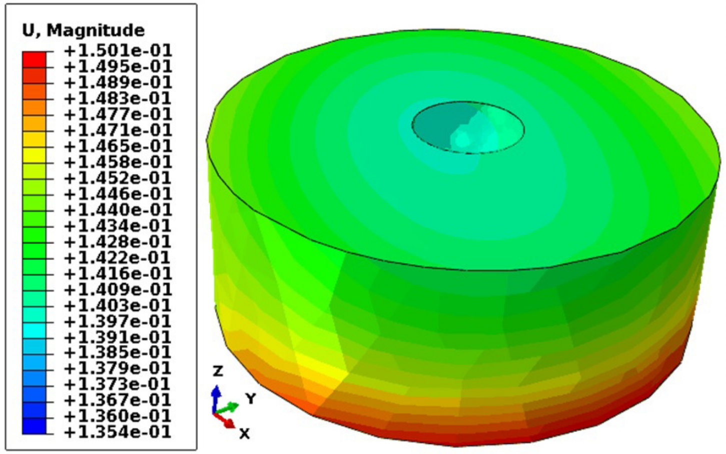
$$w = a_5\sin(\theta) + a_6\sin(2\theta) + a_7\sin(3\theta)$$

  - **Shear fields**

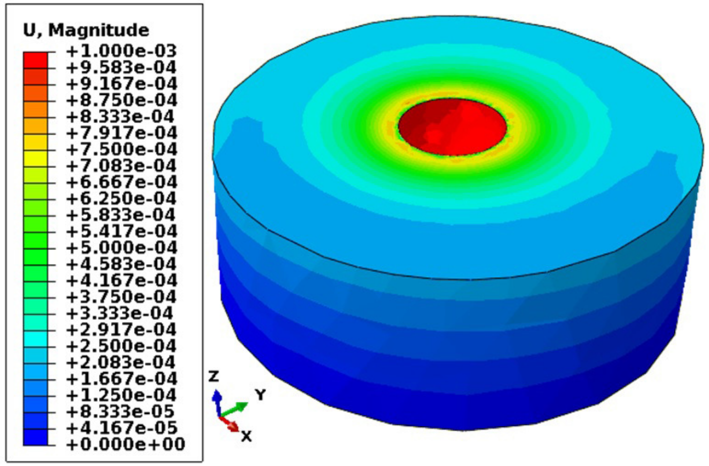
    'unperfect' loading



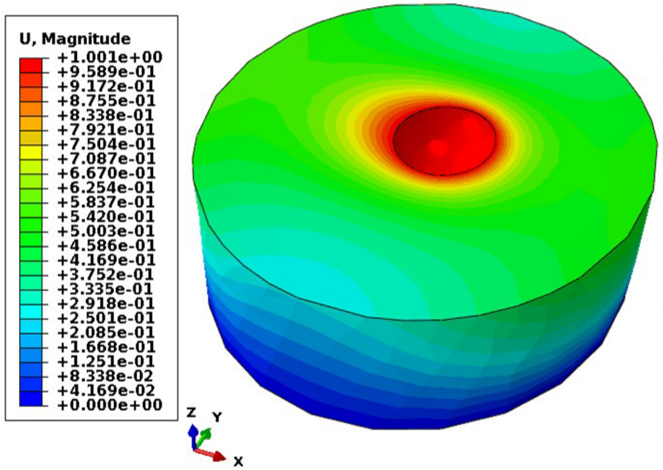
Reduced Kinematic Basis



Rigid body motion



Displacement field associated with 1st amplitude

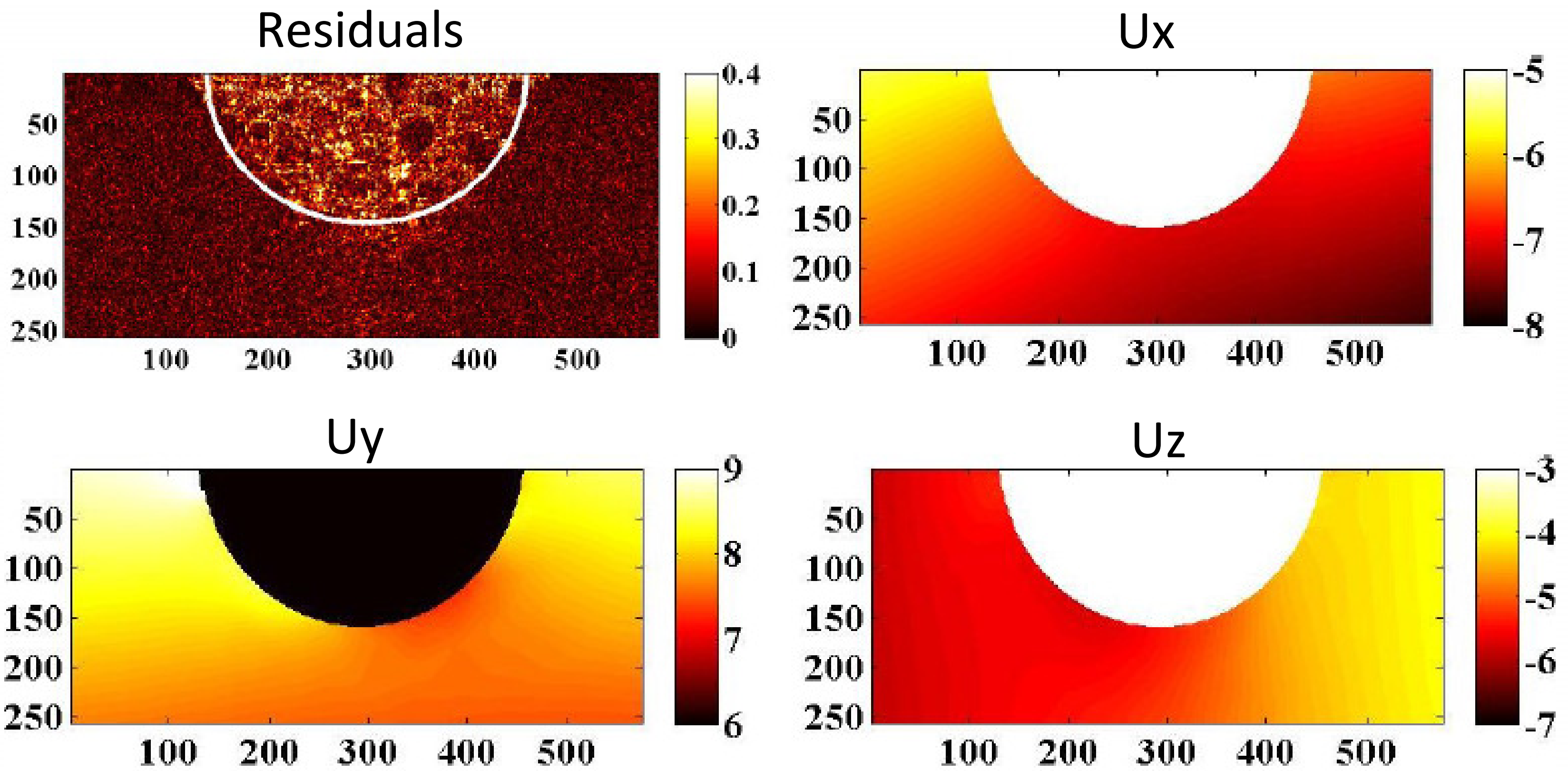


Displacement field associated with 8th amplitude

[Bouterf *et al.*, 2014, *Strain* 50 pp. 444-453]

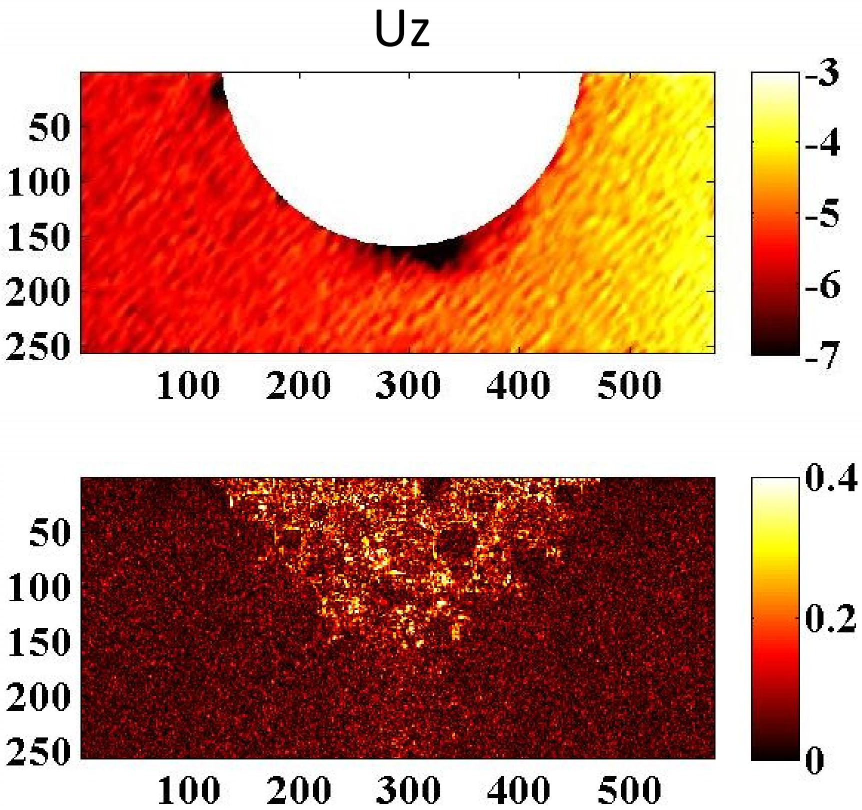
Measurement Results

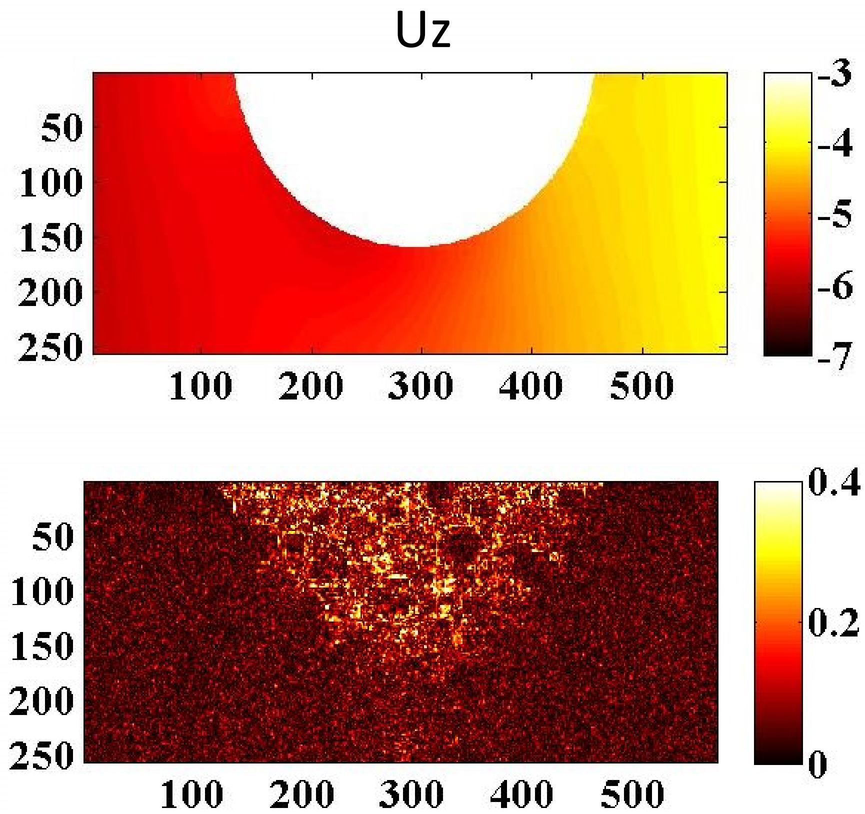
- Standard displacement resolution ≈ 0.02 voxel (ℓ = 8 voxels)
  - Divided by 20 wrt. standard C8-DVC
  - # DOF = 9



1 voxel↔12 µm

Comparison

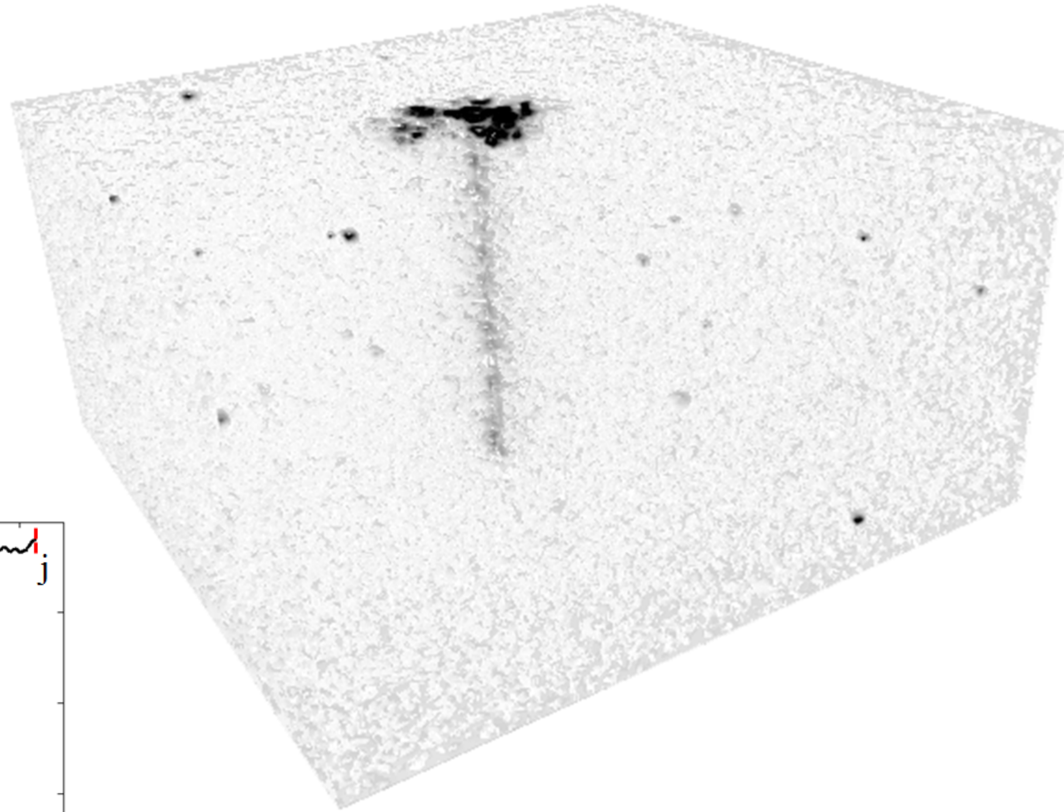


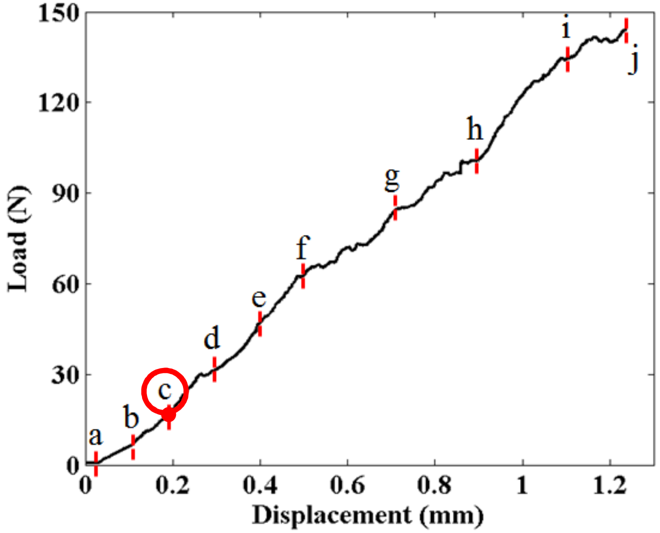


RMS GL residual: 5.06%

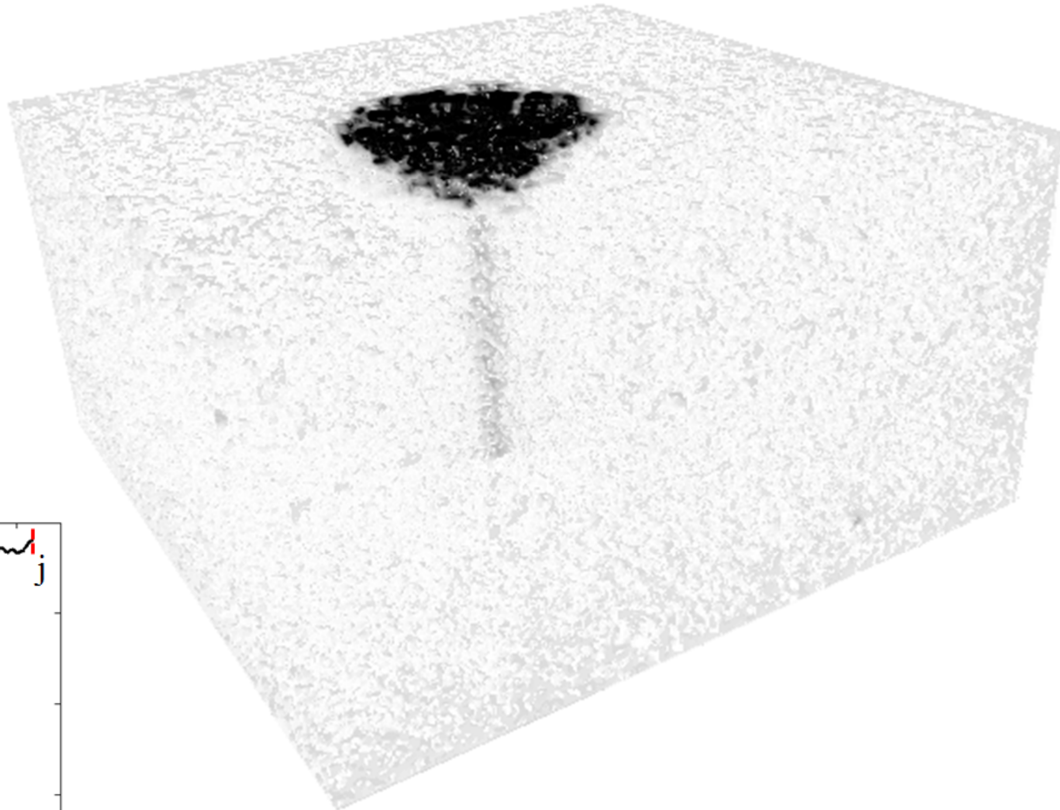
RMS GL residual: 5.14%

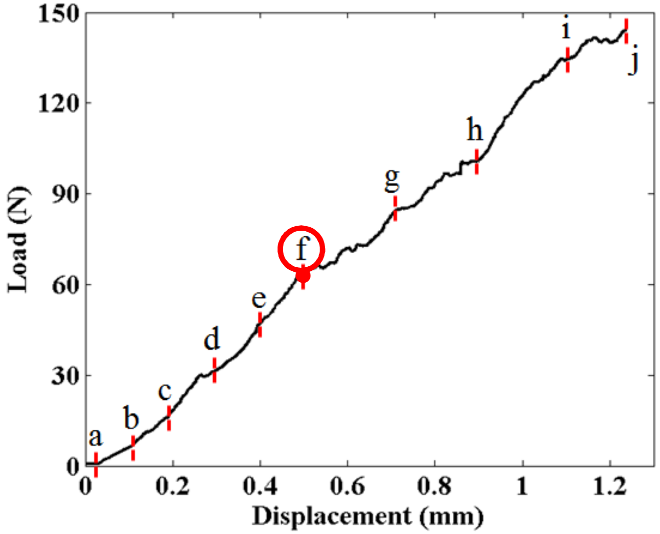
Correlation Residuals



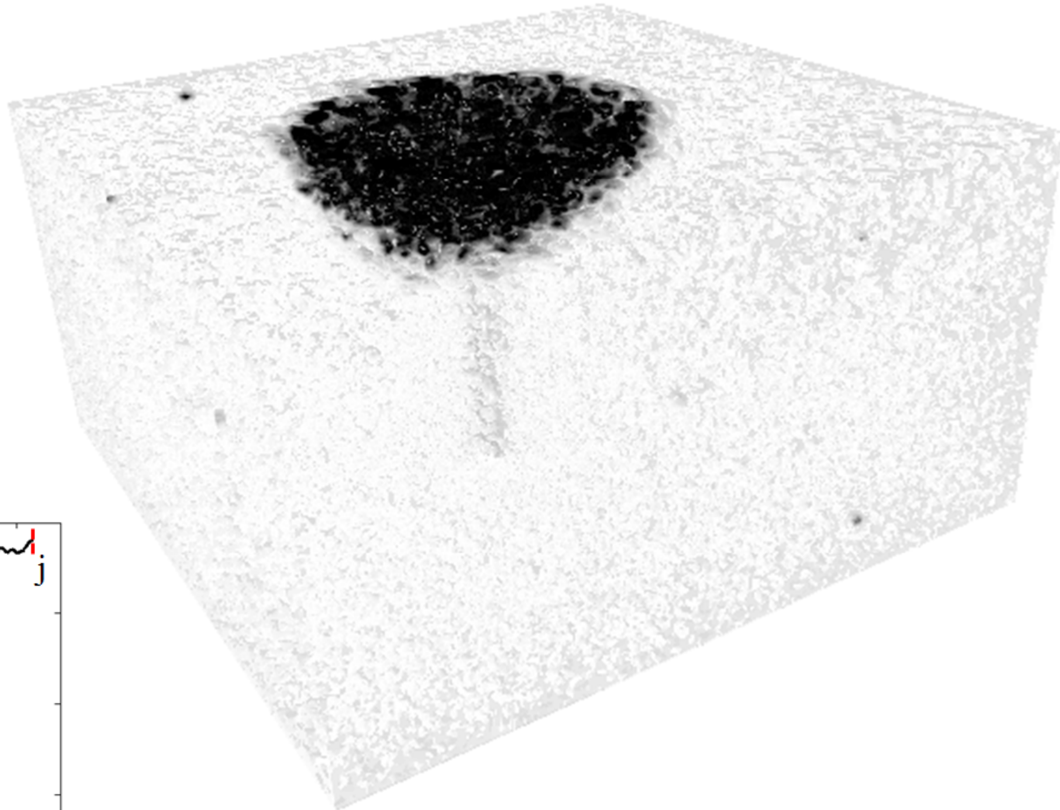


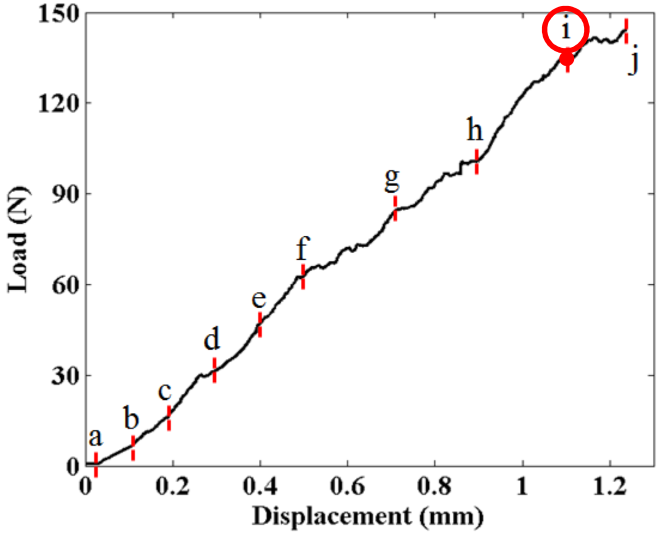
Correlation Residuals



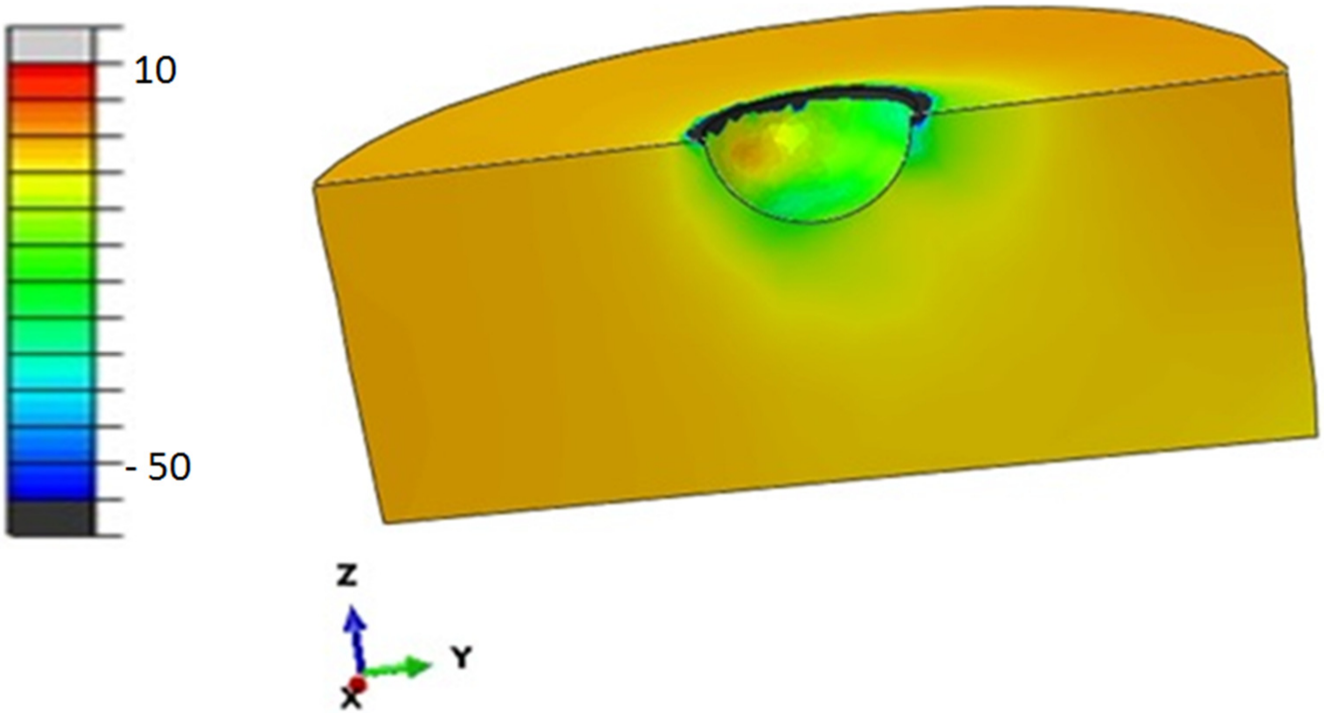


Correlation Residuals

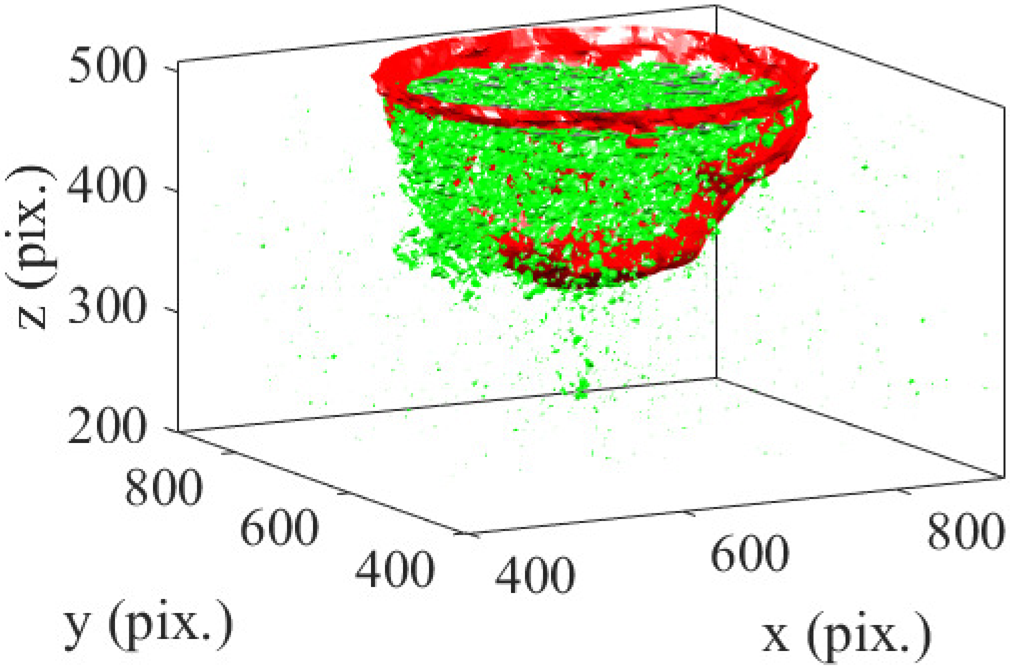




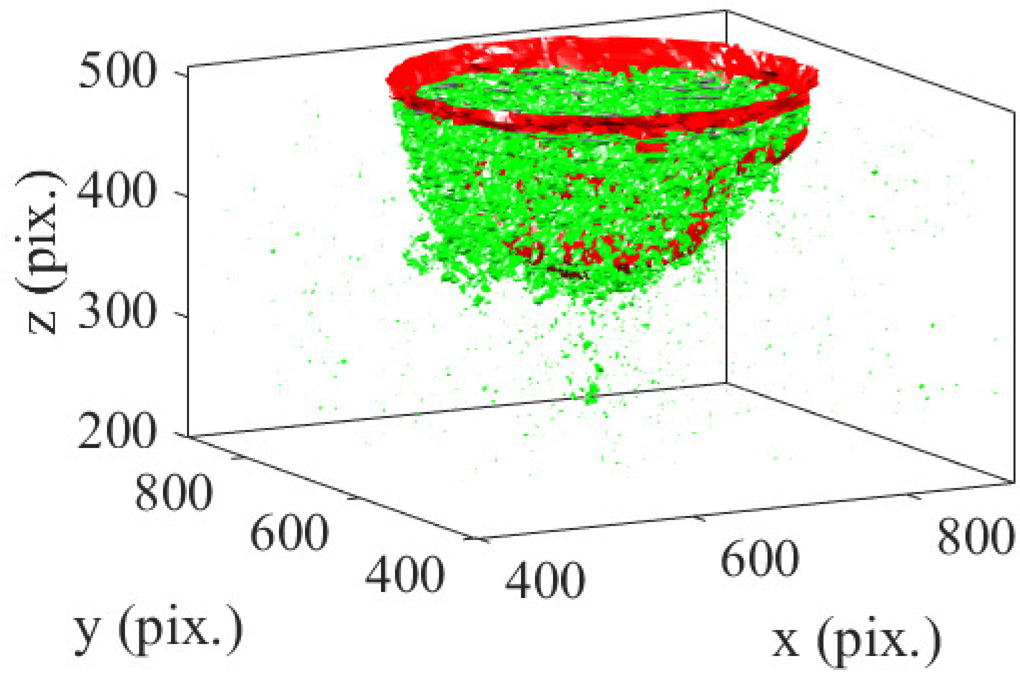
Minor Principal Stress



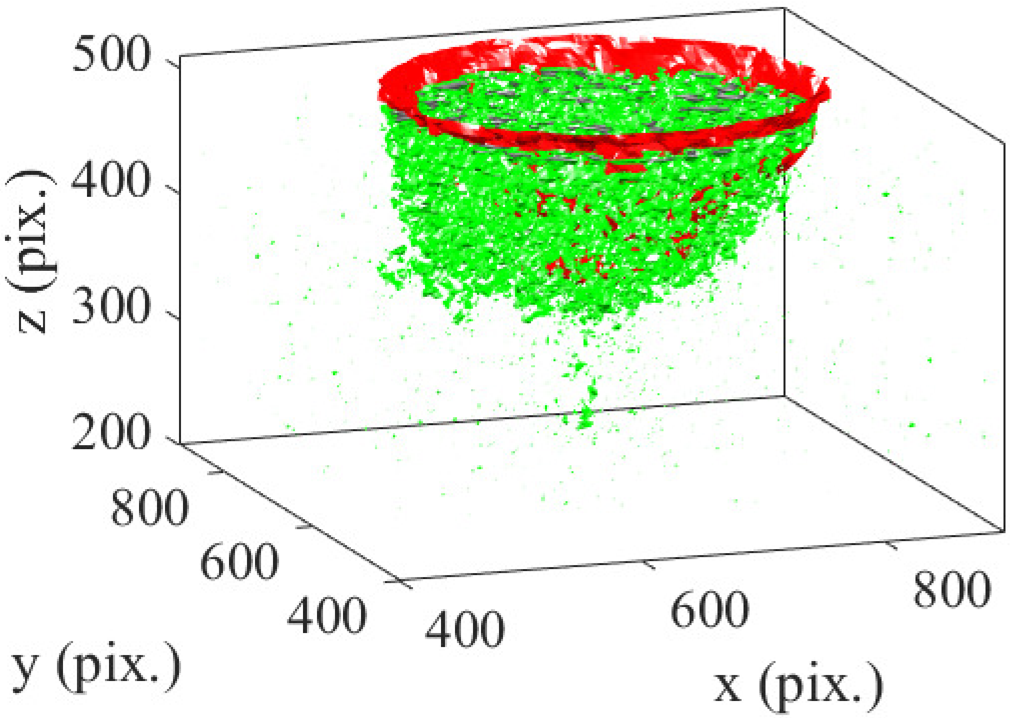
Surface Contour $\sigma_3 = -5$ MPa



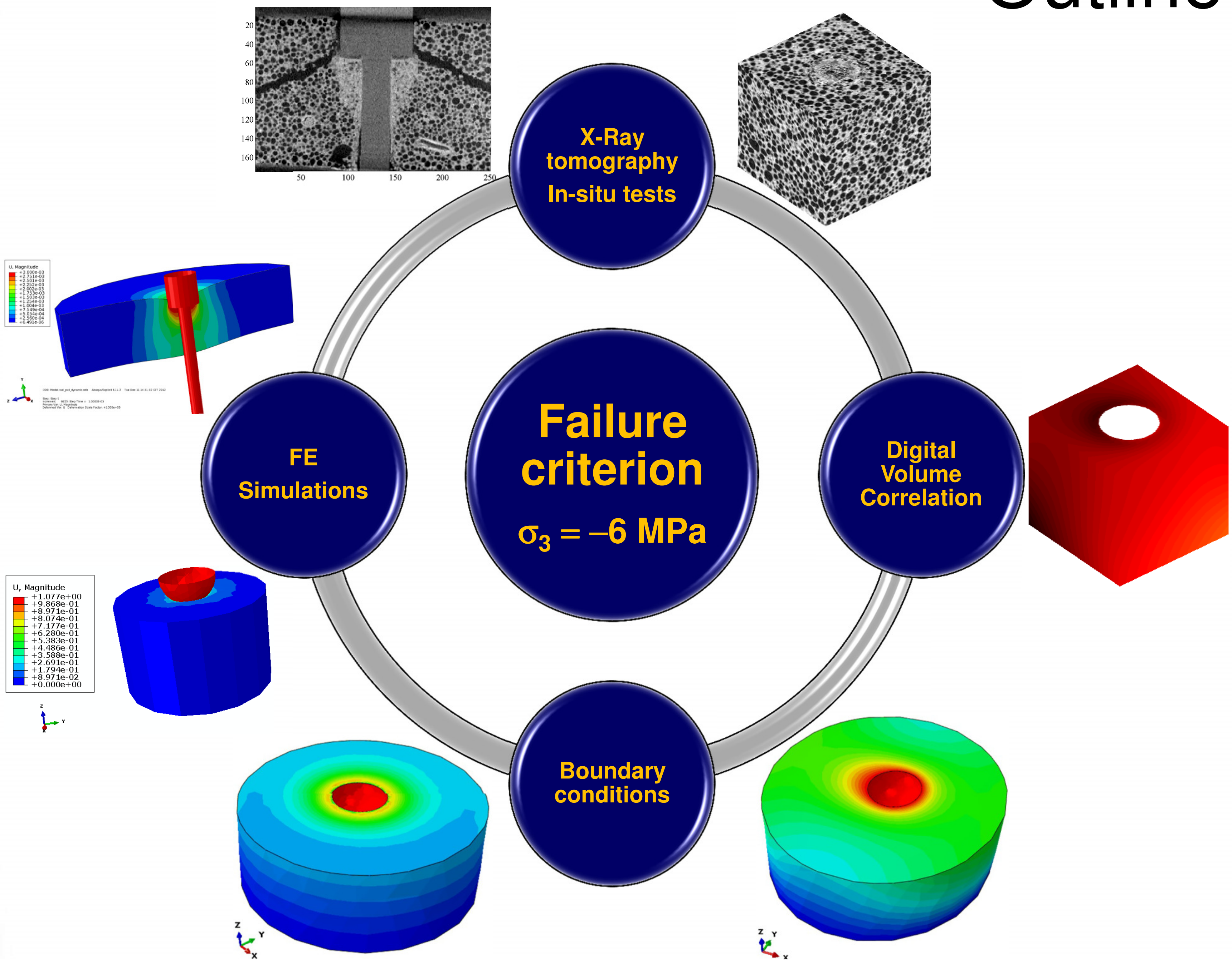
Surface Contour $\sigma_3 = -6$ MPa



Surface Contour $\sigma_3 = -7$ MPa



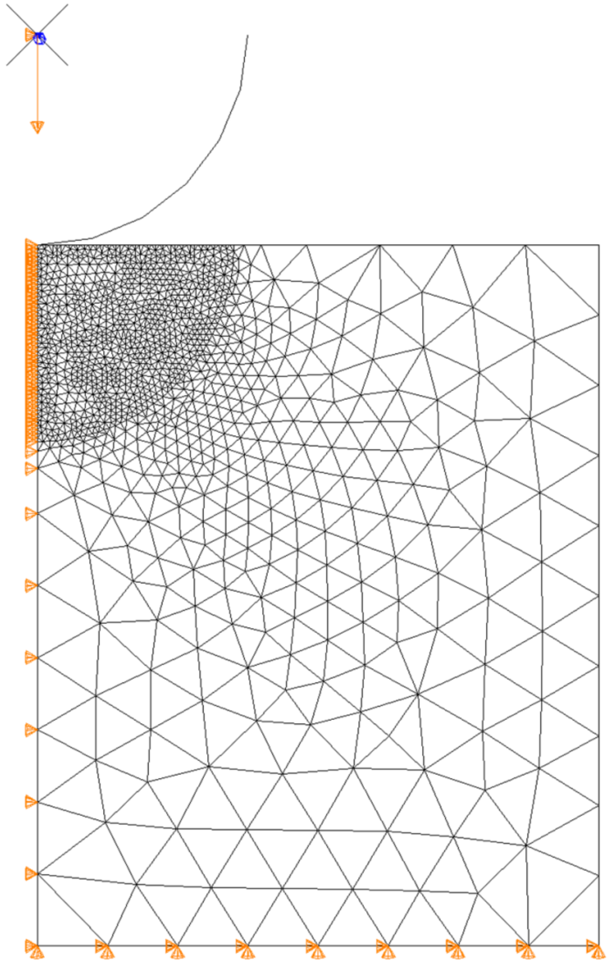
Outline

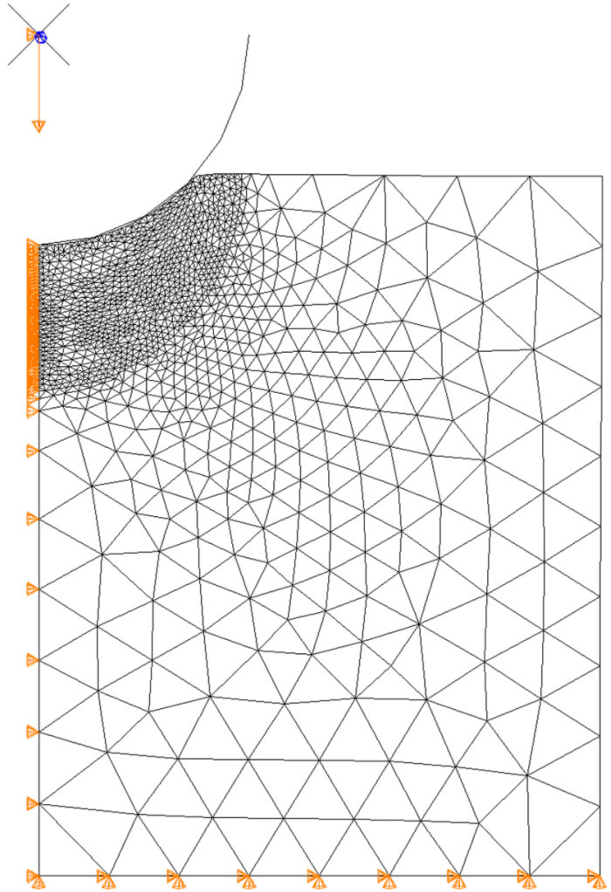


X-Ray tomography
In-situ tests
FE Simulations
Failure criterion
$\sigma_3 = -6$ MPa
Digital Volume Correlation
Boundary conditions
U, Magnitude

FE Simulations

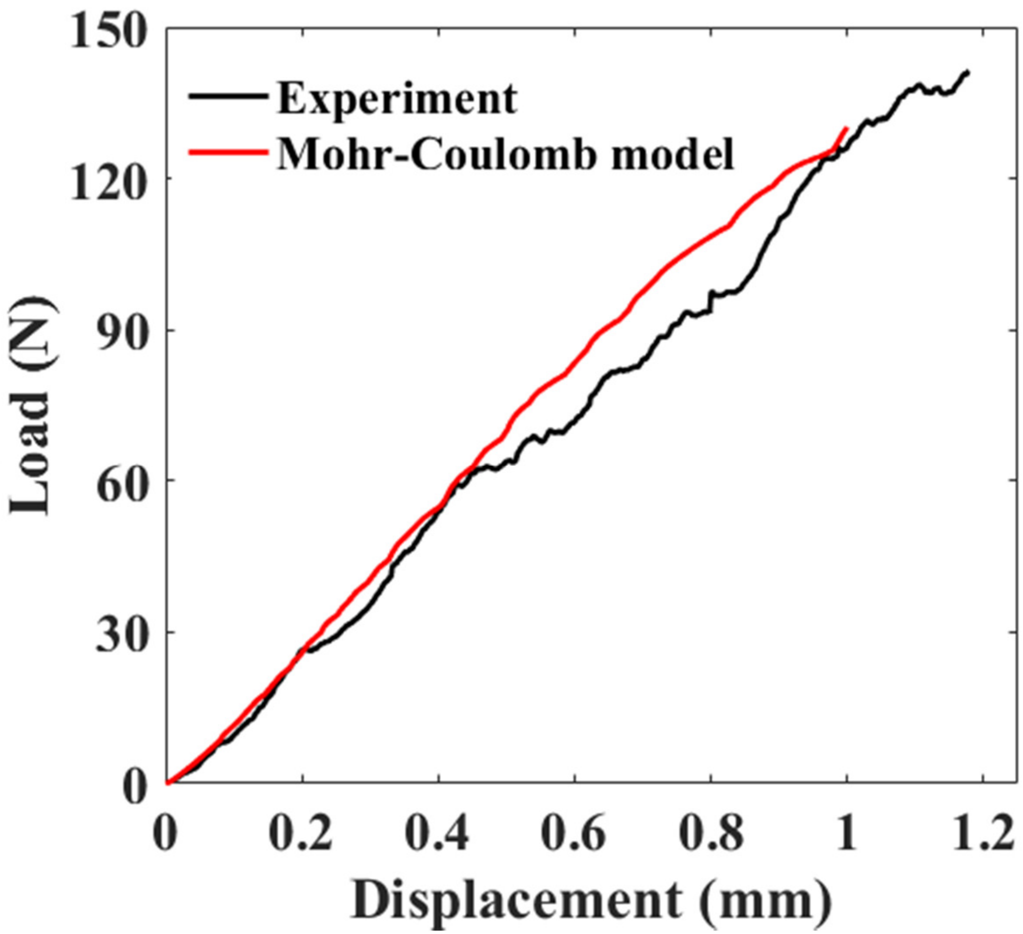
Mohr-Coulomb model



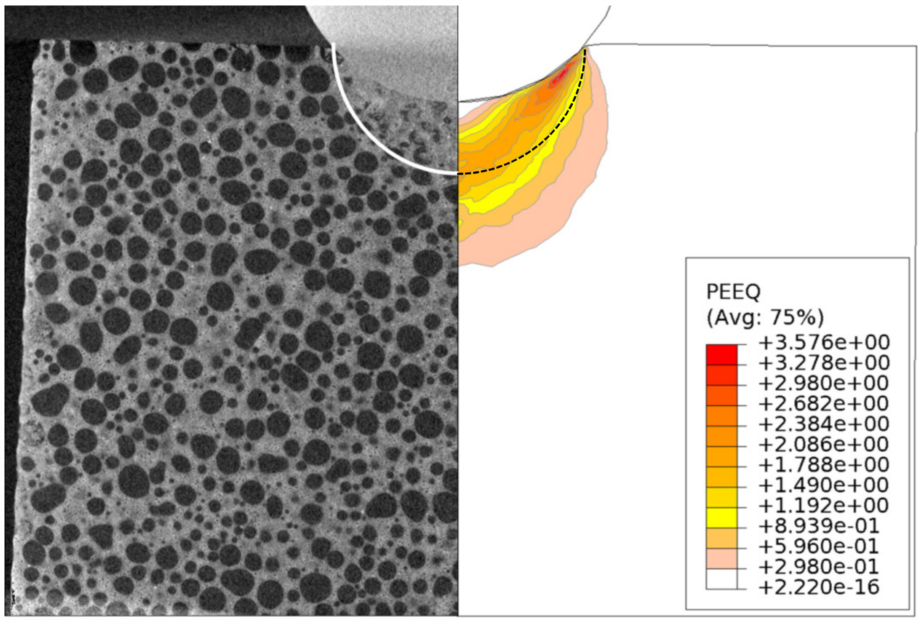


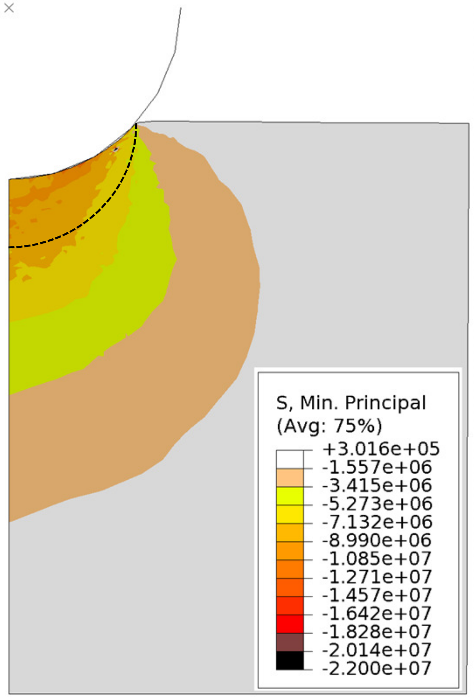
FE Simulations

Good macroscopic agreement



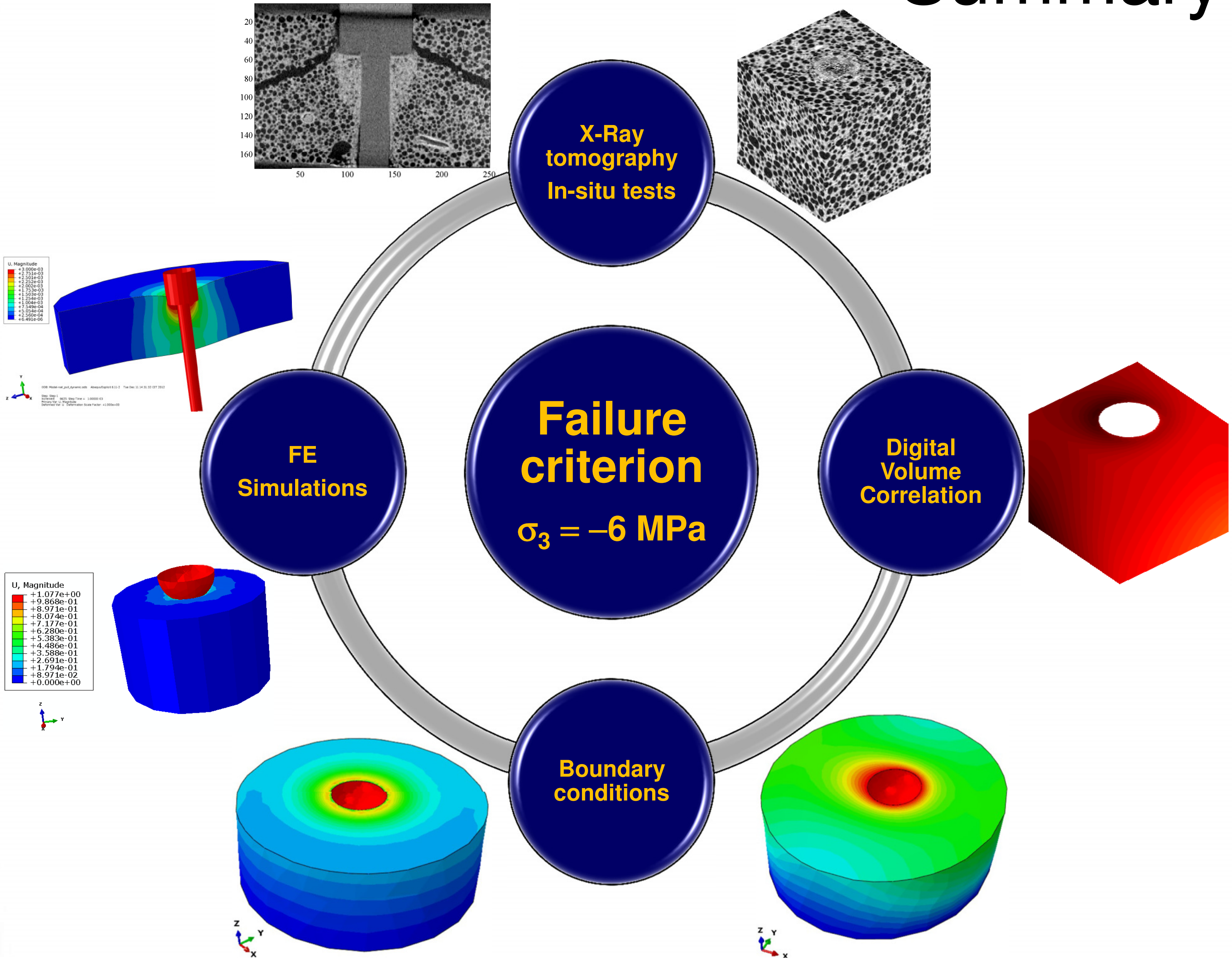
FE Simulations





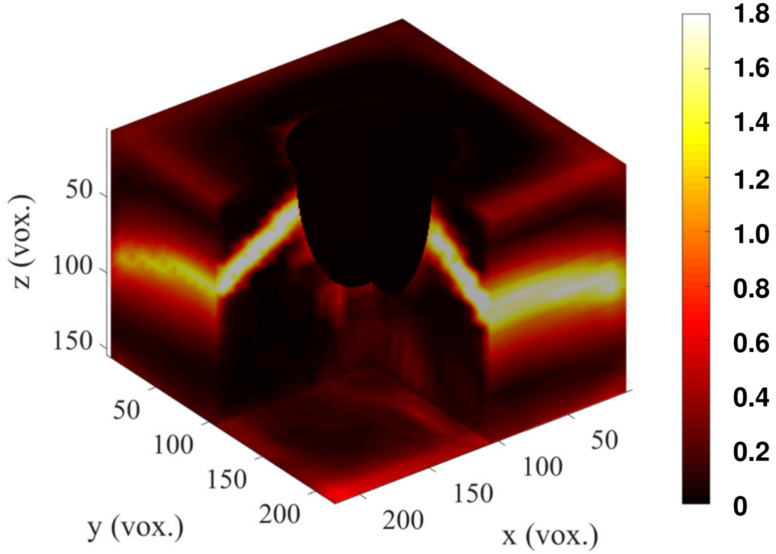
Abrupt / smooth transition

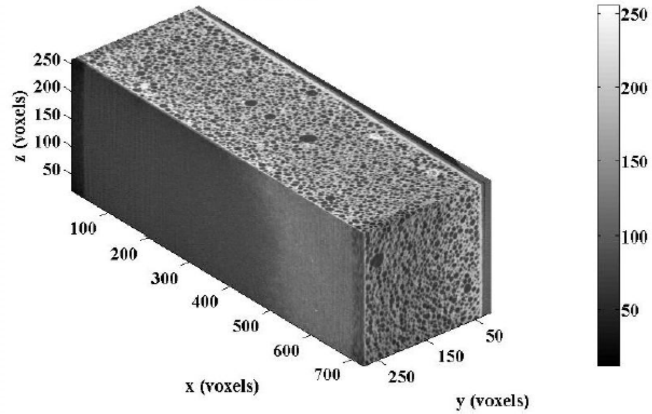
Summary


X-Ray tomography
In-situ tests
FE Simulations
Failure criterion
σ3 = –6 MPa
Digital Volume Correlation
Boundary conditions
20 40 60 80 100 120 140 160
50 100 150 200 250
U, Magnitude
+1.077e+00
+9.868e-01
+8.971e-01
+8.074e-01
+7.177e-01
+6.280e-01
+5.383e-01
+4.486e-01
+3.588e-01
+2.691e-01
+1.794e-01
+8.971e-02
+0.000e+00

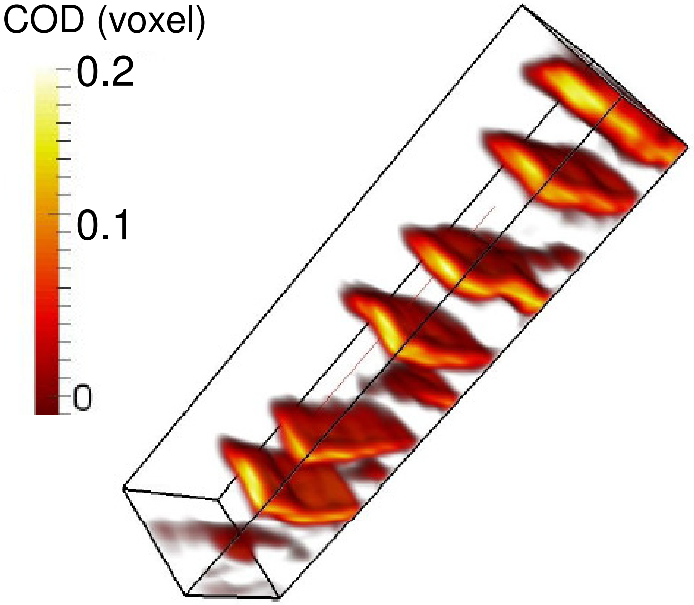
Outlook: Cracks (of course)!

COD field (voxel)

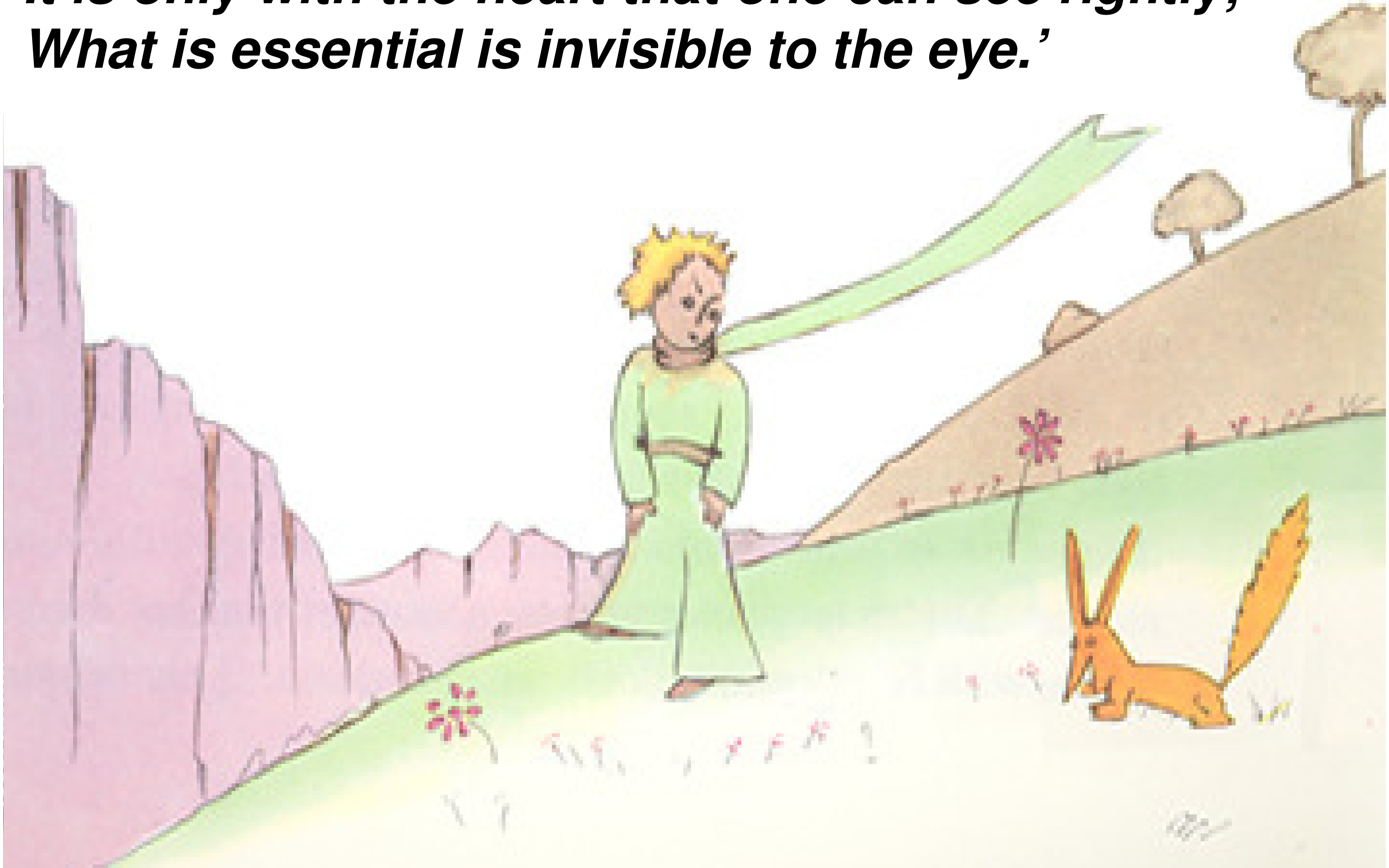




COD (voxel)



'Here is my secret. It is very simple:
It is only with the heart that one can see rightly;
What is essential is invisible to the eye.'



[A. de Saint-Exupéry, 1943, *The Little Prince*, Reynal & Hitchcock]